CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

0775 Further Mathematics 1

JUNE 2016 ADVANCED LEVEL

Centre Number	
Centre Name	
Candidate Identification No.	
Candidate Name	

Mobile phones are NOT allowed in the examination room

MULTIPLE CHOICE QUESTION PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed **Advanced Level – 0775 Further Mathematics 1.**
4. Fill in the information required in the spaces above.
5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil: **Candidate Name, Exam Session, Subject Code, Centre Number and Candidate Number.** Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.

How to answer the questions in this examination

6. Answer **ALL** the **50** questions in this Examination. All questions carry equal marks..
7. Calculators are allowed.
8. Each question has FOUR suggested answers: **A, B, C** and **D**. Decide which answer is appropriate. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square bracket for the answer you have chosen.

For example, if C is your correct answer, mark C as shown below:

[A] [B] [~~C~~] [D]

9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet using the blank spaces in the question booklet.
12. **At the end of the examination, the invigilator shall collect the answer sheet first and then the question booklet. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

Turn over

1. The integrating factor for the differential equation $x\frac{dy}{dx} + 4y = \frac{e^x}{x}$ is

A. $4x$

B. e^{4x}

C. x^4

D. $e^{\frac{4}{x}}$

2. $e^{-2i\theta} \equiv$

A. $e^{-2}(\cos\theta + i\sin\theta)$

B. $e(\cos 2\theta + i\sin 2\theta)$

C. $\cos 2\theta - i\sin 2\theta$

D. $2(\cos\theta - i\sin\theta)$

3. $\frac{x^2}{x^2-4}$ expressed in partial fractions, where P, Q and R are real constants is

A. $\frac{P}{x+2} + \frac{Q}{x-2}$

B. $P - \frac{Q}{x-2} - \frac{R}{x+2}$

C. $\frac{P}{x-2} - \frac{Q}{(x-2)^2}$

D. $P + \frac{Q}{x-2} + \frac{R}{(x-2)^2}$

4. If $f(x) = 3\cosh 2x - 1$, then the minimum value of $f(x)$ is

A. -1

B. 1

C. -2

D. 2

5. If P and Q are statements, then $\sim(P \Rightarrow Q) \equiv$

A. $P \vee Q$

B. $P \wedge \sim Q$

C. $\sim P \wedge \sim Q$

D. $\sim P \wedge Q$

6. The value of k for which

$$p(x) = \begin{cases} k(x-2), & x = 0,1,2. \\ 0, & \text{otherwise} \end{cases}$$

is a discrete probability density function is

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{3}$

D. $\frac{1}{3}$

7. The series $\sum_{r=1}^{\infty} \frac{1}{r^k}$ is divergent for

A. $k > 1$

B. $k \geq 2$

C. $k \leq 1$

D. $0 < k \leq 2$

8. $\int \frac{1}{\sqrt{x^2 + \lambda^2}}\,dx =$, where λ and k are constants is

A. $\sin^{-1}\left(\frac{x}{\lambda}\right) + k$

B. $\cosh^{-1}\left(\frac{x}{\lambda}\right) + k$

C. $\sinh^{-1}\left(\frac{x}{\lambda}\right) + k$

D. $\cos^{-1}\left(\frac{x}{\lambda}\right) + k$

9. Given that $y = |x| + x - x|x|$, $x < 0$, then $y =$

A. $2x + x^2$

B. $-x^2$

C. x^2

D. $2x - x^2$

10. $x = at^2$, $y = 2at$, $a \in \mathbb{R}$, are the parametric equations of a curve where t is the parameter. The length of the arc for $0 \le t \le 1$ is given by

A. $2a\int_0^1 \left(\sqrt{1+t}\right) dt$

B. $2a\int_0^1 \left(\sqrt{1+t^2}\right) dt$

C. $2a\int_0^1 \left(1+t\right) dt$

D. $2a\int_0^1 \left(\sqrt{1-t^2}\right) dt$

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11. The polar equation of the curve $x^2 + y^2 = 2x$ is

A. $r^2 = 2\cos\theta$

B. $r = 2\sin\theta$

C. $r^2 = 2\sin\theta$

D. $r = 2\cos\theta$

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12. If the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$ are perpendicular to each other, then the value of b is

A. 1

B. 2

C. 3

D. 4

---

13. The rate of decay of cells in a certain culture is directly proportional to the number n of cells present at time t. Treating n as a continuous variable, k a positive constant, then n satisfies the differential equation

A. $\frac{dn}{dt} = kn$

B. $\frac{dn}{dt} = -n$

C. $\frac{dn}{dt} = n$

D. $\frac{dn}{dt} = -kn$

---

14. A particle P moves on the curve with equation $r = ae^{\theta}$ with constant angular speed ω. The transverse component of acceleration of P is

A. 0

B. $-\omega^2 r$

C. $2\omega^2 r$

D. ωr

---

15. The forces $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ act at the points with position vectors $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{r}_3$ respectively. Given that $\sum_{i=1}^{3} \mathbf{F}_i \neq 0$ and $\sum_{i=1}^{3} \mathbf{r}_i \times \mathbf{F}_i = 0$ then this system of forces

A. is in equilibrium

B. reduces to a couple

C. reduces to a single force through O

D. reduces to a single force and a couple.

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16. The diagram shows the components u and v of velocity (perpendicular and parallel to a fixed vertical wall respectively) of an elastic ball just before and after impact.

If e is the coefficient of restitution, then

A. $e = 0$

B. $e = 1$

C. $e > 0$

D. $e < 1$

---

17. Given that

$(\lambda\mathbf{i} - 4\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - \mathbf{k}) = -4\mathbf{i} + 8\mathbf{j} + 16\mathbf{k}$

where $\lambda \in \mathbb{R}$, the value of λ is

A. 0

B. 4

C. 8

D. 16

18. If w and z are complex numbers then the transformation $w = \frac{1}{2}z$ is an

A. enlargement centre $(0,0)$, factor $\frac{1}{2}$

B. enlargement centre $(0,\frac{1}{2})$, factor $\frac{1}{2}$

C. enlargement centre $(\frac{1}{2},0)$, factor $\frac{1}{2}$

D. enlargement centre $(\frac{1}{2},\frac{1}{2})$, factor $\frac{1}{2}$

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19. Given that $a \oplus b = a + b - 1$, where $\oplus$ is a binary operation on $\mathbb{R}$, the inverse of an element $x \in \mathbb{R}$ under $\oplus$ is

A. x

B. $x-2$

C. $2-x$

D. $1-x$

---

20. Given that $1553^{25} \equiv x(\bmod 7)$, then $x =$

A. 1

B. 3

C. 4

D. 6

---

21. Given that $\frac{dy}{dx} + y = 0$ and $y = 1$, when $x = 0$, a quadratic approximation for y is

A. $1 - x + \frac{x^2}{2}$

B. $1 + x - \frac{x^2}{2}$

C. $1 - x - \frac{x^2}{2}$

D. $1 + x + \frac{x^2}{2}$

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22. A compound pendulum oscillates through small angles θ about its equilibrium position such that $8a\dot{\theta}^2 = 9g\cos\theta$, $a > 0$. Its period is

A. $2\pi\sqrt{\frac{8a}{9g}}$

B. $\frac{3\pi}{8}\sqrt{\frac{a}{g}}$

C. $2\pi\sqrt{\frac{9g}{8a}}$

D. $\frac{8\pi}{3}\sqrt{\frac{a}{g}}$

---

23. X is a random variable such that $E(X) = 2$, then $E(3X-4) =$

A. 18

B. 14

C. 2

D. 6

---

24. Let P and Q be statements. The compound statement $\otimes$ on the truth table could represent

P	Q	$\otimes$
T	T	T
T	F	F
F	T	T
F	F	T

A. $P \Leftrightarrow Q$

B. $P \Rightarrow Q$

C. $P \wedge Q$

D. $P \vee Q$

---

25. Given that $f(x) = \begin{cases} x^2+1, & x < 1 \\ \frac{1-2x}{x+2}, & 1 \le x \le 3 \\ x-8, & x > 3 \end{cases}$

For what value(s) of x is $f(x)$ discontinuous?

A. 1 and 2

B. 1 and 3

C. 2 and 3

D. 2 only

---

26. If x_0 and y_0 is a solution of the Diophantine equation $ax + by = c$ and $\gcd(a,b) = d$, then for $t \in \mathbb{Z}$, all the solutions are of the form

A. $x = x_0 + \frac{b}{d}t$ and $y = y_0 + \frac{a}{d}t$

B. $x = x_0 + \frac{a}{d}t$ and $y = y_0 + \frac{b}{d}t$

C. $x = x_0 - \frac{a}{d}t$ and $y = y_0 - \frac{b}{d}t$

D. $x = x_0 + \frac{b}{d}t$ and $y = y_0 - \frac{a}{d}t$

27. The inverse of the permutation

$P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ is

A. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

28. Which one of the equations represents a polar curve that is symmetrical about the initial line $\theta = 0$?

A. $r = a\sin 3\theta$
B. $r = 3a\sin\theta$
C. $r = a\sin\theta\cos 2\theta$
D. $r = 3a\cos\theta$

29. Let $f(x) = \dfrac{1}{(x+2)(x-1)^2}$. If

$f(x) \equiv \dfrac{1}{9(x+2)} - \dfrac{1}{9(x-1)} + \dfrac{P}{(x-1)^2}$, then the value of the constant P is

A. $-\frac{1}{3}$
B. -3
C. $\frac{1}{3}$
D. 3

30. If $z = 1 + \cos\theta + i\sin\theta$, then zz^*, where z^* is the conjugate of z is

A. $4\cos^2\frac{\theta}{2}$
B. $4\sin^2\frac{\theta}{2}$
C. $\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$
D. 2

31. Given that the series $\sum_{n=1}^{\infty} n^p$ converges, then

A. $p \geq 1$
B. $p < -1$
C. $0 \leq p < 1$
D. $-1 \leq p < 0$

32. The moment of inertia of a uniform rod of mass $2m$ and length l about an axis through its centre and perpendicular to the rod is

A. $\dfrac{2ml^2}{3}$
B. $\dfrac{ml^2}{4}$
C. $\dfrac{ml^2}{6}$
D. $\dfrac{ml^2}{8}$

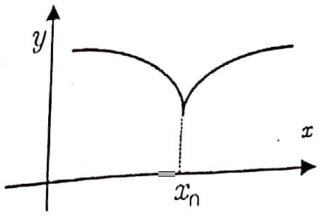
33. $\dfrac{d}{dx}\left[\ln(\cosh 3x)\right] =$

A. $3\,\text{cosech}\,3x$
B. $3\,\text{sech}\,3x$
C. $3\tanh 3x$
D. $3\coth 3x$

34. $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \dfrac{n-1}{n} I_{n-2}$, $I_5 =$

A. $\frac{2}{3}$
B. $\frac{4}{5}$
C. $\frac{1}{2}$
D. $\frac{8}{15}$

35. Given the sketch of the curve of $y = f(x)$



Which of the following statements is NOT true?

A. $\lim_{x \to x_0} f(x) = f(x_0)$

B. $\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ exists

C. $\lim_{x \to x_0^+} f(x) = f(x_0)$

D. $\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x)$

36. The general solution of the differential equation $\frac{d^2x}{dt^2} + x = t$, where p and q are constants is $x =$

A. $p\cos t + q\sin t + 2t$

B. $p\cos t + q\sin t - 2t$

C. $p\cos t - q\sin t + t$

D. $p\cos t + q\sin t - t$

37. An estimate for $\int_2^4 \frac{1}{x}\,dx$, using Simpson's rule with 3 ordinates is

A. $\frac{25}{36}$

B. $\frac{17}{36}$

C. $\frac{25}{24}$

D. $\frac{17}{24}$

38. The velocities of a sphere before and after impact are $\mathbf{i} + 2\mathbf{j}$ and $3\mathbf{i} + \mathbf{j}$ respectively. The angle of deflection due to the collision is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{5}$

D. $\frac{\pi}{6}$

39. A liquid is leaking from a container at the rate of $\frac{2}{x}$ litres per second, where x is the quantity in litres of the liquid remaining in the container after time t seconds. The time in which the quantity of the liquid reduces from 20 litres to 10 litres in seconds is

A. 500

B. 300

C. 100

D. 75

40. Given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$, the value of y when $x = 0.1$, using a step length of 0.1 and the approximation

$y_{n+1} \cong y_n + h\left(\frac{dy}{dx}\right)_n$ is

A. 1

B. 1.1

C. 1.22

D. 1.362

41. The equations of the asymptotes of the hyperbola $x^2 - 6x - 4y^2 + 16y - 8 = 0$ are

A. $x + 2y = 0,\ x - 2y = 0$

B. $x + 2y + 4 = 0,\ x - 2y + 3 = 0$

C. $x + 2y + 2 = 0,\ x - 2y - 2 = 0$

D. $x + 2y - 7 = 0,\ x - 2y + 1 = 0$

42. Given that the angle between the vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + p\mathbf{j} + \mathbf{k}$ is $\frac{\pi}{4}$, then the value of p is

A. $\sqrt{2}$

B. $\sqrt{3}$

C. 2

D. $\sqrt{6}$

43. The general solution of the differential equation $\frac{dy}{dx} + 2xy = y$, where c is a constant is

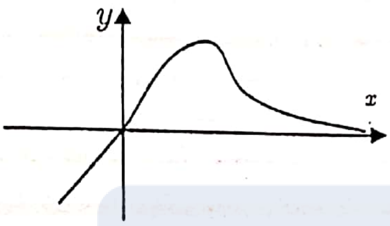
A. $y = e^{x-x^2} + c$

B. $y = ce^{x^2-x}$

C. $y = ce^{x-x^2}$

D. $y = e^{x^2-x} + c$

44. Which of the following equations best suits the graph



A. $y = xe^{x}$

B. $y = xe^{-x}$

C. $y = -xe^{-x}$

D. $y = -xe^{x}$

45. The transformation matrix $\begin{pmatrix} 1 & 2 & 0 \\ -2 & -4 & -2 \\ 1 & 2 & 1 \end{pmatrix}$ maps the xyz space onto

A. a line

B. a plane

C. the origin

D. space

46. The invariant complex number under the transformation $w = \frac{z-i}{1+i}$ is

A. -1

B. $-i$

C. i

D. 1

47. Which of the following series is convergent ?

A. $\sum_{r=1}^{\infty}(-1)^r$

B. $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{n+1}{n}\right)$

C. $\sum_{k=0}^{\infty}\frac{1}{3^k+1}$

D. $\sum_{n=1}^{\infty}\left(n^2+n\right)$

48. $\text{arsinh}(x-1) =$

A. $\ln\left[(x-1)+\sqrt{x^2-2x+2}\right]$

B. $\ln\left[(x-1)+\sqrt{x^2-2x}\right]$

C. $\ln\left[(x-1)+\sqrt{x^2+2x+2}\right]$

D. $\ln\left[(x-1)+\sqrt{x^2+2x}\right]$

49. If $[x]$ is the "greatest integer" function, then $\int_1^4 [x-1]\,dx =$

A. 2

B. 3

C. 4

D. $\frac{9}{2}$

50. Given the operation table for a binary operation $\otimes$, the inverse of X is

A. W

B. X

C. Y

D. Z

$\otimes$	W	X	Y	Z
W	Y	Z	X	W
X	Z	Y	W	X
Y	X	W	Z	Y
Z	W	X	Y	Z

STOP

GO BACK AND CHECK YOUR WORK