Pure Maths With Mechs 2
765

# CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

## General Certificate of Education Examination

**JUNE 2015** **ADVANCED LEVEL**

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| Subject Title | **Pure Mathematics With Mechanics** |
| Paper No. | **Paper 2** |
| Subject Code No. | **765** |

**Two and a half hours.**

**Full marks may be obtained for answers to ALL questions.**

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

**Start each question on a fresh page.**

1. (i) Given that $(x-1)$ is a factor of the polynomial $f(x)$, where $f(x) = ax^4 + x^3 - 12x^2 - x + 2$.
Find the value of the constant $a$ and verify that $f(-1) = 0$.

(ii) Find the value of the constant $k$ for which the equation

$$x^2 + (k+1)x + k = 0$$

has one root double the other.

**(9 marks)**

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2. (i) Show that $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$.

(ii) Find the general solution of the equation

$$\sin 4x + \cos 2x = 0.$$

(iii) Solve for $x$, where $0^\circ \le x \le 180^\circ$, the equation $\sin 3x + \cos x = 0$.

**(10 marks)**

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3. (i) The function $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x - \frac{x^3}{3}$.
Find the monotony of $f$, showing clearly its variation table.

(ii) Solve the differential equation $x\frac{dy}{dx} = y(2x^2 + 1)$.

**(12 marks)**

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4. (i) Given that $z = e^{i\theta}$, show that $z^n + z^{-n} = 2\cos n\theta$.
Use this result to express $\cos^5\theta$ in terms of cosines of multiples of $\theta$.

(ii) Given that $z_1 = 1 + i\sqrt{3}$ and $z_2 = -1 + i$, evaluate
(a) $|z_1 z_2|^2$,
(b) $\arg(z_1{}^4)$.

**(11 ma**

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5. The coordinates of the points $A$, $B$ and $C$, are $(0, 1, 3)$, $(-1, 0, 1)$ and $(1, -1, 2)$ respectively.
Find
(a) $\overrightarrow{AB} \times \overrightarrow{BC}$,
(b) the sine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$,
(c) the value of the constant $\mu$ for which the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mu\mathbf{i} - \mathbf{j} + 5\mathbf{k})$ is parallel to the plane containing $A$, $B$ and $C$.

**(10 marks)**

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6. Given the matrix A, where $A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$,
find
(a) det(A), the determinant of A,
(b) $A^{-1}$, the inverse of A.

Hence, or otherwise, solve the system of equations,

$$2x + y + 2z = 3,$$
$$3x + y + 2z = 3,$$
$$2x + 2y + z = 2.$$

**(8 marks)**

7. Express $\frac{1}{(3t+1)(t+1)}$ in partial fractions.

By using the substitution $t = \tan x$, or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \frac{dx}{3 + 5\sin 2x} = \frac{1}{8}\ln 2.$$

**(9 marks)**

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8. (i) Find the set of real values of $x$ for which $\left|\frac{3x+4}{2x-3}\right| < 1$.

(ii) Sketch the curve of $y = \frac{x+2}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$, showing clearly the intercepts with the coordinate axes and the behaviour of the curve as it approaches its asymptotes.

**(11 marks)**

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9. (i) The functions f and g∘f are defined by

$g : x \mapsto x + 5, x \in \mathbb{R}$, $g \circ f : \mapsto \frac{6(x-3)}{x-4}$, $x \in \mathbb{R}, x \neq 4$,

find f and show that f is injective.

(ii) If p is the statement : *Eric plays golf* and q the statement: *Oscar plays tennis*.
Write down the statement represented by each of the following:
(a) $p \Rightarrow q$,
(b) $-q \Rightarrow p$,
(c) $-(p \vee q)$.

**(11 marks)**

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10. (i) The first three terms in the series expansion of $\sqrt{\frac{1-x}{1+kx}}$ are 1, $-2x$ and $4x^2$ respectively.
Determine the value of $k$ and state the range of values of $x$ for which the expansion is valid.

(ii) Five cards are to be dealt out to a player from a standard pack of 52 playing cards . How many different possibilities are there if,
(a) there is no ace
(b) there are at least two aces.

**(11 marks)**

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