

Pure Math With Stats 3

0770/3

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2017

ADVANCED LEVEL

Subject Title	Pure Mathematics With Statistics
Paper No.	3
Subject Code No.	0770

Three hours

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and tables produced by the GCE Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Electronic calculators may be used.

Start each question on a fresh page.

1. The probability mass function of a discrete random variable X is defined by

$$f(x) = \begin{cases} \frac{1+kx}{22} & , x = 0, 1, 2, 3, \\ 0 & , \text{otherwise.} \end{cases}$$

- Obtain the value of the constant k .
- Write down the probability distribution of X .
- Find $E(X)$ and $Var(X)$
- Calculate the values of $E(11X - 4)$ and $Var(11X - 4)$.

2. Three events A, B and C are defined in a sample space S .

Given that $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{5}$, $P(C) = \frac{1}{2}$ and $P(B \cap C) = \frac{1}{10}$,

Find (a) $P(A \cup B)$, if A and B are independent.

(b) $P(C/B)$,

(c) $P(C/\bar{B})$,

(d) the probability that one and only one of the events B or C will occur.

3. The marks obtained by some 50 class six pupils in an aptitude test into secondary schools are shown in the frequency distribution below.

Marks, x	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25
Frequency, f	14	9	11	10	6

Calculate, to two decimal places,

- the mean and the standard deviation of this distribution,
- the mode and the median of the marks distribution.

4. The probability that a woman from a particular area has brown eyes is $\frac{2}{5}$.

A random sample of 100 women is to be selected from this area.

Using the normal distribution as an approximation to the binomial distribution, estimate to three decimal places, the probability that

- at least half of the women will have brown eyes.
- the number of women who will have brown eyes will be between 30 and 45 inclusive.
- exactly 35 women will have brown eyes.

5. The continuous random variable X has probability density function, f , where

$$f(x) = \begin{cases} kx(2-x), & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Find (a) the value of the constant k ,
- (b) the mean and variance of X ,
- (c) the cumulative distribution function of X .

6. State two properties of the Poisson distribution.

In a certain malaria ward, the mean number of malaria parasites detected per milliliter of blood from the patients is 4. Find to three decimal places, the probability that in a randomly extracted one milliliter of blood, there will be

- no malaria parasite,
- four malaria parasites,
- less than three malaria parasites.

Find, also, the probability that in another randomly extracted one-third milliliters of blood, there will be no malaria parasite.

7. (i) The probability that a basket ball player will make a basket in any trial is $\frac{1}{4}$.

Find the probability that

- she makes her first basket in her fourth trial.
- she requires at most 5 trials to make her first basket.

- (ii) A large population has mean, $E(X) = \mu$.

- (a) Independent identical random samples X_1, X_2 and X_3 are drawn from this population.

Show that $\hat{\mu}$ is an unbiased estimator of the population mean, where

$$\hat{\mu} = \frac{1}{2}X_1 + \frac{1}{5}X_2 + \frac{3}{10}X_3.$$

- (b) A random sample of size 27 from this population produced the following statistics:

$$\sum x = 1560 \text{ and } \sum (x - \bar{x})^2 = 168900.$$

Calculate to two decimal places, the most efficient unbiased estimate of the population mean and the population variance.

8. For 10 towns in a certain country, the heights above sea level, x , in hundreds of metres, and their temperatures, y , in degree centigrade on a particular day were recorded as shown in the table below.

City	A	B	C	D	E	F	G	H	I	J
Height, x	18	11	3	5	8	11	4	15	16	5
Temperature, y	9	13	18	17	13	10	16	10	6	14

Calculate, to three decimal places,

- the Pearson product-moment correlation coefficient for this data and interpret your result.
- the least squares regression line of temperatures on heights.
- the temperature of a city that is 1300m above sea level on that particular day.