Pure Math With Stats 2
770/2

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2017 **ADVANCED LEVEL**

Subject Title	Pure Mathematics With Statistics
Paper No.	2
Subject Code No.	770

Three hours

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

1. (i) Find the set of values of k for which the roots of the quadratic equation $x^2 - kx + 2x + k - 2 = 0$, are real and different.

   (ii) Given that $2^x = 3^y$ and that $x + y = 1$, show that $x = \frac{\log 3}{\log 6}$. **(7 marks)**

2. (i) The first term of an arithmetic progression is a and the common difference is -1. If the sum of the first n terms is equal to the sum of the first $3n$ terms of this progression, express a in terms of n.
   Obtain the value of a when $n = 10$ and hence find the sum of the first 30 terms of the progression.

   (ii) Find the position of the term in x^{-12} in the expansion of $\left(x^3 - \frac{1}{x}\right)^{24}$. **(11 marks)**

3. (i) (a) Find $\frac{dy}{dx}$ if $y = \left(\sqrt{1 + 2x^2}\right)^5$.

   (b) Given that $y = \ln\left(\frac{1+x^2}{1-x^2}\right)$, show that $\frac{dy}{dx} = \frac{4x}{1 - x^4}$.

   (ii) The parametric equations of a curve are given by
   $x = ct$ and $y = \frac{c}{t}$, where t is a parameter and c is a constant.
   Show that an equation of the tangent to the curve at the point P with parameter t is given by

   $x + t^2 y = 2ct$. **(10 marks)**

4. (i) The table below shows corresponding values of x and y which approximately satisfy a relation of the form $y = an^x$, where a and n are constants.

x	2	3	4	5	6
y	13.6	27.2	54.4	108.8	217.6

   By drawing a suitable linear graph, determine the values of and n, correct to one decimal place.

   (ii) Given that $x = 0.2$ is a first approximation to the root of the equation $f(x) = 0$, where

$$f(x) = x^3 + 3x - 1,$$

   use one iteration of the Newton – Raphson procedure to obtain a second approximation to the root of the equation, giving your answer to two decimal places. **(11 marks)**

5. (i) Given that $(x + 1)$ and $(x - 2)$ are factors of the expression $ax^3 - x^2 + bx - a$, find the values of the constants a and b.
   Hence, find also the remainder when the expression is divided by $(x - 4)$.

   (ii) Express as a single fraction $\frac{5}{x+2} - \frac{7}{2x+3}$, simplifying the numerator. **(8 marks)**

6. (i) The functions f and g are defined by $f : x \mapsto \frac{3}{x - 2}$, $x \in \mathbb{R}$, $x \neq 2$,

$$g : x \mapsto \frac{x - 1}{x + 2}, x \in \mathbb{R}, x \neq -2.$$

   (a) Find $f \circ g(x)$ and $g \circ f(x)$, stating their domains.
   (b) Show that g is not surjective.

   (ii) A relation R is defined on the set $A = \{1, 2, 3, 4, 5\}$ by
   $a R b \Leftrightarrow a + b = 2n, n \in \mathbb{N}$.
   List all the equivalence classes of A under R. **(9 marks)**

$f(x) = \frac{3}{x-2}$ $g(x) = \frac{x-1}{x+2}$ $f \circ g(x)$

7. (i) Find all the values of θ, $0 \leq \theta \leq 2\pi$ for which

$$\sin 2\theta = \sec\theta.$$

(ii) Show that the matrix $\mathbf{M}$ is invertible, where

$$\mathbf{M} = \begin{pmatrix} 3 & -2 & 5 \\ 7 & 4 & -8 \\ 5 & -3 & -4 \end{pmatrix}.$$

Hence, find $\mathbf{M}^{-1}$, the inverse of $\mathbf{M}$. **(12 marks)**

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8. (i) Given that, $\left(\frac{4 - 3i}{2 - i}\right) z - (1 + 3i) = 1 - 2i$,

express the complex number z in the form $a + bi$, where a and b are real constants.

(ii) The vector equations of two lines L_1 and L_2 are given by

$$L_1:\ \mathbf{r} = 13\mathbf{i} + 4\mathbf{j} + 11\mathbf{k} + \lambda(3\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}),$$

$$L_2:\ \mathbf{r} = 5\mathbf{i} + 22\mathbf{j} + 9\mathbf{k} + \mu(7\mathbf{i} - 17\mathbf{j} - 5\mathbf{k}).$$

Find

(a) the position vector of the point of intersection of L_1 and L_2,

(b) the cosine of the acute angle between L_1 and L_2. **(12 marks)**

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9. (i) The equations of two circles S_1 and S_2 are given by

$$S_1:\ x^2 + y^2 + 2x + 2y + 1 = 0,$$

$$S_2:\ x^2 + y^2 - 4x + 2y + 1 = 0.$$

Show that S_1 and S_2 touch externally and obtain the equation of the common tangent T at the point of contact.

(ii) A father and a mother have 5 children. This family is to occupy a particular front-line bench in church on a special thanksgiving service.

Given that this bench has a capacity of 7 persons, in how many ways can this family be seated on the bench

(a) if the parents must seat adjacent to each other.

(b) if the parents must seat adjacent to each other at one end of the bench. **(10 marks)**

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10. (i) Given that

$$f(x) \equiv \frac{2}{x(x+1)(x+2)},$$

express $f(x)$ in partial fractions.

Hence show that $\int_2^4 f(x)\,dx = \ln\left(\frac{27}{25}\right)$.

(ii) Sketch the graph of the curve whose equation is given by $y = \frac{2x - 7}{x - 4}$, showing clearly the points where the curve meets the coordinate axes and the behavior of the curve near its asymptotes. **(12 marks)**