CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

0770 MATH WITH STATISTICS 2

JUNE 2018 ADVANCED LEVEL

Subject Title	Mathematics With Statistics
Paper No.	2
Subject Code No.	0770

THREE HOURS

Full marks may be obtained for answers to ALL questions.

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

1. $P(x) = ax^3 - 3x^2 + bx + 6$.
When $P(x)$ is divided by $(x - 1)$, the remainder is -6. Given that $(x + 2)$ is a factor of $P(x)$, find the values of the constants a and b. Hence, solve the equation
$$P(x) = 0.$$
(9 marks)

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2. (i) Given that one root of the quadratic equation $x^2 - 8x + k = 0$ is three times the other, find the value of the constant k. Hence, solve the equation
$$x^2 - 8x + k = 0.$$
(ii) A relation R is defined on the set of integers $\mathbb{Z}$ by
$a\,R\,b$ if '$a + 2b$' is a multiple of 3.
Show that R is an equivalence relation.
(4 , 5 marks)

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3. (i) Differentiate with respect to x,
(a) $\frac{ln(1+x^2)}{x^5}$,
(b) $\sin^2(x + 1)$.
(ii) Given that $f(x) = x^3 - x^2 - x + 5$,
find the set of values of x for which $f(x)$ is increasing.
(5 , 6 marks)

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4. (i) Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of x, stating the range of values of x for which the expansion is valid.
(ii) Find the value of x for which
$y = 2\log_2 x$ and $y + 4 = \log_2 2x$.
(5 , 3 marks)

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5. (i) The complex number z is given by
$$z = \frac{3-i}{2+i}.$$
Express z in the form $a + bi$, where $a, b \in \mathbb{R}$.
(ii) Given that $z = \cos\theta + i\sin\theta$, show that
$$z^3 + z^{-3} = 2\cos 3\theta$$
Hence, find the general solution of the equation
$$z^3 + z^{-3} = \sqrt{3}.$$
(3 , 6 marks)

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6. Express $\frac{x}{(x+1)(x+2)}$ in partial fractions.
Solve the differential equation
$$(x + 1)(x + 2)\frac{dy}{dx} = x(y + 1),$$
for $x > -1$, given that $y = \frac{1}{2}$ when $x = 1$, expressing the solution in the form $y = f(x)$.

(10 marks)

7. (i) Find the coordinates of the centre and the length of the radius of the circle
$$x^2 + y^2 - 3x - 4 = 0.$$
Show that the line $3x + 4y - 17 = 0$ is a tangent to the circle.

(ii) $\mathbf{M}$ is a 3×3 matrix given by
$$\mathbf{M} = \begin{pmatrix} 3 & 4 & 0 \\ 1 & x+2 & 1 \\ 2x-4 & 4 & x-4 \end{pmatrix}.$$
Given that $\mathbf{M}$ is a singular matrix, show that
$$3x^2 - 2x - 36 = 0.$$

(7 , 5 marks)

8. (i) Find the general solution of the equation
$$\sin\left(x + \frac{\pi}{6}\right) = 2\cos x.$$

(ii) The vector equations of two lines L_1 and L_2 are

$$L_1: \mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k})$$

$$L_2: \mathbf{r} = -\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}).$$

Find
(a) the point of intersection of L_1 and L_2.
(b) the vector parametric equation of the plane containing L_1 and L_2.

(4 , 5 marks)

9. (i) Show that the equation $x\ln x + x - 3 = 0$ has a root between 1 and 2.
Given that $x = \frac{3}{2}$ is a first approximate root, use one iteration of the Newton-Raphson procedure to obtain a second approximate root of this equation.

(ii) A class is made up of 5 boys and 8 girls. Find the number of ways in which a mixed delegation of 4 students can be chosen from the class if it must include at least 2 boys.

(iii) Two statements p and q are defined as follows:
p: the workers will go on strike.
q: there will be no salary.
Write the following statements in correct English.
(a) $q \Rightarrow p$
(b) $\sim q \Rightarrow \sim p$
(c) $p \wedge q$.

(5,4 , 3 marks)

10. (i)

x	2	6	10	14	16
y	62	270	580	994	1248

The table above gives the values of a continuous variable y for some observed values of x.
It is known that y and x are connected by a law of the form

$$y = ax^2 + bx, \text{ where } a, b \in \mathbb{R}.$$
By drawing a suitable linear graph, estimate the values of a and b, giving the answers, correct to two decimal places.

(ii) The first and last terms of an arithmetic progression are 7 and 51 respectively.
Given that the sum of the terms of the progression is 348, find the number of terms and the common difference of this progression.

(8 , 3 marks)