Further Mathematics 3
0775

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2016 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	3
Subject Code No.	0775

Two and a half hours

Answer ALL questions.

For your guidance the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. a) The forces $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j} - 6\mathbf{k})$N and $\mathbf{F}_2 = (-7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$N act through the points with position vectors $(-10\mathbf{i} - 5\mathbf{j} - 54\mathbf{k})$m and $(-30\mathbf{i} - 24\mathbf{k})$m , respectively. The forces $\mathbf{F}_3 = (2\mathbf{i} + \mathbf{j} + \mathbf{k})$N and $\mathbf{F}_4 = (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$N act through the point with position vector $(-10\mathbf{j} + 6\mathbf{k})$m
Show that the system of the four forces is in equilibrium. **(8 marks)**

b) The force $\mathbf{F}$ of magnitude 42 N acts in the direction of the vector $(5\mathbf{i} - 4\mathbf{j} + 20\mathbf{k})$m . A particle P of mass 5 kg moves under the action of $\mathbf{F}$ from the point $A(2, 1. - 4)$ to the point $B(-3, -7, 10)$. Find the work done by $\mathbf{F}$. **(5 marks)**

2. Consider a population P at time t with net relative growth rate k and constant emigration rate m.
a) Show that the time rate of change of P is modeled by the differential equation

$$\frac{dP}{dt} = kP - m$$

where k and m are positive constants. **(2 marks)**

b) Find the general solution of this differential equation, given that $P = P_0$ when $t = 0$. **(5 marks)**

c) Show that

(i) if $m = kP_0$, then the population $P(t)$ is constant **(2 marks)**

(ii) for $m > kP_0$, the population $P(t)$ is declining. **(3 marks)**

3. (a) An aircraft starts from rest and taxis along the runway before taking off. The table below shows how the acceleration a m s^{-2} of the aircraft varies with time t seconds.

t	0.00	5.0	10.0	15.0	20.0	25.0	30.0
a	0.05	0.25	0.40	0.60	0.80	1.05	1.35

Use Simpson's rule to find:

(i) the speed, in m s^{-1}, of the aircraft after 1.5 seconds. **(5 marks)**

(ii) the distance, in metres, travelled by the aircraft in 1.5 seconds. **(6 marks)**

(b) A function y satisfies the differential equation

$$\frac{dy}{dx} + y^2 = x^2.$$

Given that $y = 1$ when $x = 0$ and that $y = 0.91$ when $x = 0.1$, use the approximation

$$2h\left(\frac{dy}{dx}\right)_n \approx y_{n+1} - y_{n-1},$$

and a step length of 0.1 to find the value of y when $x = 0.4$, giving your final answer correct to 3 decimal places. **(4 marks)**

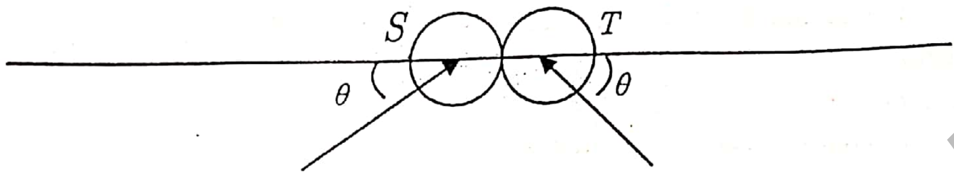
4. (a) In a certain competitive examination, a candidate commits an average of 1 error in every 25 questions answered. Use the Poisson distribution to find, to 3 significant figures, the probability that in 100 questions answered, there are

(i) no errors committed, **(4 marks)**

(ii) at most 4 errors committed. **(3 marks)**

(b) The masses of grapes from a particular orchard are normally distributed with mean μ and variance σ^2. Given that 5% of the grapes have masses greater than 80g and 10% have masses less than 20g, find the values of μ and σ giving your answers correct to 1 decimal place. **(6 marks)**

5. A smooth sphere S of mass λm, moving with speed λu collides with a smooth sphere T of equal radius but of mass m moving with speed u. At impact, the direction of motion of each sphere makes an acute angle θ with the line of centres as shown below.



Given that the coefficient of restitution between the spheres is $1/2$, and that after impact, the direction of motion of S makes an angle 2θ with the line of centres, show that

(i) the speed of S along the line of centres is $\frac{1}{2}u(2\lambda - 3)\cos\theta$ **(6 marks)**

(ii) $1 + \tan^2\theta = \frac{3}{\lambda}$ **(4 marks)**

(iii) $0 < k < 3$. **(4 marks)**

6 A thin uniform rod AB of mass $3m$ and length $5a$ is free to rotate in a vertical plane about a smooth horizontal axis through the end A. The rod is released from rest when AB is horizontal. When the rod makes an angle θ with the downward vertical through A, its angular speed is $\sqrt{\frac{2g}{5a}}$.

(i) Show that $\theta = \cos^{-1}\left(\frac{2}{3}\right)$. **(7 marks)**

(ii) Find the angular velocity of the system as the rod first passes through the vertical position. **(3 marks)**

7. A particle P of mass m moves in a straight line under the action of a force of magnitude $13m|x|$ directed towards a fixed point O, where x metres is the distance of P from O. The resistance to motion of P is of magnitude $6m|v|$, where v is the speed of P.

Show that the equation of motion of P is $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 0$. **(2 marks)**

Initially, P is 3 metres away from O and moving towards O with speed 3 m/s. Express x in the form $x = Ae^{-3t}\sin(2t + \lambda)$, giving the values of A and λ. **(9 marks)**

Find the period of oscillation. **(2 marks)**

8. A bead B, moves along a smooth wire in the form of a curve whose polar equation is $r = \lambda(1 + \cos\theta)$, where $\lambda > 0$, in such a way that the transverse component of the acceleration is zero. Show that:

(i) $r^2\frac{d\theta}{dt} = \text{constant}$. **(3 marks)**

(ii) Show also that the radial component of the acceleration in terms of r and θ is $(\lambda - 2r)\left(\frac{d\theta}{dt}\right)^2$ **(7 marks)**