for more free gce questions, notes and free answers, visit http://www.gcerevision.com or download our app "kawlo" on playstore

FURTHERMATHS 2
0775

# CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

**JUNE 2018** **ADVANCED LEVEL**

| | |
|---|---|
| Subject Title | Further Mathematics |
| Paper No. | 2 |
| Subject Code No. | 0775 |

**THREE HOURS**

**INSTRUCTIONS TO CANDIDATES**

**Answer ALL 10 questions.**

*For your guidance, the approximate mark allocation for parts of each question is indicated.*

*Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.*

*In calculations, you are advised to show all the steps in your working, giving your answer at each stage.*

# REPLACEMENT

gcerevision.com

for more free gce questions, notes and free answers, visit http://www.gcerevision.com or download our app "kawlo" on playstore



1. Given the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4e^{-x}$$

Find

i) the complementary function in the form $y = f(x)$. **(3 marks)**

ii) the particular integral in the form $y = g(x)$ **(3 marks)**

iii) the solution of the differential equation for which $y = 0, \frac{dy}{dx} = 1$ when $x = 0$. **(4 marks)**

---

2. The position vectors of four non-coplanar points $A, B, C$ and $D$ relative to the origin, $O$, are $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ respectively, where

$$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$$
$$\mathbf{b} = \mathbf{i} + 6\mathbf{j} - \mathbf{k}$$
$$\mathbf{c} = 5\mathbf{i} + \mathbf{k}$$
$$\mathbf{d} = 3\mathbf{j} - 2\mathbf{k}$$

Find

(i) The vector equation of the plane containing $A, B$ and $C$. **(5 marks)**

(ii) the volume of the parallelpiped with vertices $A, B, C$ and $D$. **(4 marks)**

---

3. (i) Given that two roots of the complex equation

$$z^5 = 1$$

are $z = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ and $z = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}$

Find the other three roots in the form $z = \cos\theta + i\sin\theta$, $-\pi < \theta \leq \pi$ **(4 marks)**

(ii) Given also that $w = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$,

and that the structure $\left(\{1, w, w^2, w^3, w^4\}, \times\right)$

forms a group, find the identity element and the inverse of each element. **(5 marks)**

---

4. Using the definition of $\text{sech}\, x, x \geq 0$ interms of $e^x$, prove that

$$\text{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right), \quad 0 < x \leq 1$$

Hence, or otherwise, show that

(i) $\frac{d}{dx}\left(\text{sech}^{-1} x\right) = \frac{-1}{x\sqrt{1 - x^2}}$ **(3 marks)**

(ii) $\int_{\frac{1}{2}}^{1} \text{sech}^{-1} x\, dx = \frac{\pi}{3} - \frac{1}{2}\ln\left(2 + \sqrt{3}\right)$ **(6 marks)**

gcerevision.com

for more free gce questions, notes and free answers, visit http://www.gcerevision.com or download our app "kawlo" on playstore



5. Given the matrix $M$, where

$$M = \begin{pmatrix} -1 & 1 & 1 \\ 3 & h & -2 \\ h & 1 & 1 \end{pmatrix}$$

(i) Find the real values of $h$ for which $M$ is non-invertible. (3 marks)

(ii) For $h = 0$, find the image of the line

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k}),\ \lambda \in \mathbb{R}$$

under transformation $M$. (3 marks)

---

6. (i) Find, $d$, the greatest common divisor of 81 and 21. (2 marks)

(ii) Hence, find the values of $a$ and $b$, $a, b \in \mathbb{Z}$, such that

$$81a + 21b = d.$$ (3 marks)

---

7. (i) Show that the curve with polar coordinates $(r, \theta)$ where

$$r^2 = \frac{12}{3 + \sin^2\theta}$$

is an ellipse, $E$, in the $(x, y)$ plane, where

$$E: 3x^2 + 4y^2 = 12$$ (3 marks)

(ii) Find the equation of the normal to $E$ at the point $\left(1, \frac{3}{2}\right)$. (3 marks)

(iii) This normal cuts the coordinate axes at $A$ and $B$. Show that the area of triangle $OAB$ is $\frac{1}{16}$. (4 marks)

---

8. (a) Prove by contradiction that $\frac{1}{\sqrt{3}}$ is irrational. (6 marks)

(b) The function, $f$, is defined by

$$f(x) = \ln\left(\frac{1}{1 - x^2}\right),\ |x| < 1$$

Prove that the length of arc of the curve traced by $f$ for $0 \le x \le \frac{1}{\sqrt{3}}$ is

$$\ln\left(2 + \sqrt{3}\right) - \frac{1}{\sqrt{3}}$$ (7 marks)

---

gcerevision.com

for more free gce questions, notes and free answers, visit http://www.gcerevision.com or download our app "kawlo" on playstore



9. A function, $f$, is defined by

$$f(x)=\frac{2x^2+x+1}{x+1}, x\neq -1$$

Given that $\lim_{x\to\infty}\left(\frac{f(x)}{x}\right)=a$,

find

(i) the value of $a$ **(2 marks)**

(ii) $\lim_{x\to\infty}[f(x)-ax]$. **(2 marks)**

(iii) Hence, or otherwise, find the equation of the oblique asymptote to the curve $y=f(x)$. **(1 mark)**

(iv) Show that $f(x)\leq -7$ or $f(x)\geq 1$. **(3 marks)**

(v) Sketch the curve $y=f(x)$. **(3 marks)**

(vi) Find the point of symmetry of the curve $y=f(x)$. **(2 marks)**

---

10. Given three sequences $(u_n)$, $(v_n)$ and $(w_n)$ defined by

$$u_0=1,\ u_{n+1}=\frac{1}{3}(u_n+2v_n),$$

$$v_0=12,\ v_{n+1}=\frac{1}{4}(u_n+3v_n),$$

$$w_n=v_n-u_n.$$

(i) Show that $(w_n)$ is a geometric sequence with positive terms. **(2 marks)**

(ii) Find the limit of the sequence $w_n$. **(2 marks)**

(iii) Show that $u_n$ is increasing. **(2 marks)**

(iv) Show that $v_n$ is dereasing. **(2 marks)**

(v) Hence show that $u_0\leq u_n\leq v_n\leq v_0$. **(2 marks)**

(vi) Show that the sequences $u_n$ and $v_n$ each converges to $l$. **(2 marks)**

Another sequence $(t_n)$ is defined by $t_n=3u_n+8v_n$

(vii) Show that $t_n$ is a constant sequence, and hence find $l$. **(4 marks)**

gcerevision.com