FURTHER MATHEMATICS PAPER 2
0775

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2017 ADVANCED LEVEL

Subject Title	**Further Mathematics**
Paper No.	2
Subject Code No.	0775

Three hours

Answer ALL 10 questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. The points A, B, C and D have Cartesian coordinates $(1,4,3),(5,3,1),(-4,-2,3)$ and $(-1,-3,-5)$ respectively relative to the origin O.
Find
   (i) the area of triangle ABC. **(4 marks)**
   (ii) the length of the perpendicular line from the point A to the line BC. **(2 marks)**
   (iii) the Cartesian equation of the plane ABC. **(2 marks)**
   (iv) the Cartesian equation of the plane which contains the point D and is perpendicular to both the planes ABC and $\mathbf{r}.(4\mathbf{i}+\mathbf{j}-3\mathbf{k})=7$ **(2 marks)**

2. (a) Find the values of λ and μ for which the transformation T with matrix

$$M=\begin{pmatrix}1 & 2 & \mu\\ 2 & \lambda & -2\\ 3 & 6 & -3\end{pmatrix}$$

maps $\mathbb{R}^3$ onto a line, and find the equation of the line. **(4 marks)**

   (b) Find the plane which is the image of space under the matrix

$$N=\begin{pmatrix}1 & 1 & 5\\ 0 & -1 & 2\\ 1 & 3 & 1\end{pmatrix}.$$

Find the image of the plane $x+3y-z=0$. **(5 marks)**

3. (a) Using the substitution $xy=v$ where v is a function of x, transform the differential equation

$$x^2\frac{dy}{dx}+xy=y^3$$

into a differential equation in v and x. **(2 marks)**

Hence show that for $y=x=1$

$$y^2=\frac{3x}{2+x^3}.$$ **(3 marks)**

   (b) Solve completely the equation $z^5+z^3-z^2-1=0$, giving your answer in the form $z=\cos\theta+i\sin\theta$. **(4 marks)**

4. (a) Find the polar coordinates of the point(s) on the curve $r=a\sin\theta,\ 0\le\theta\le\pi$, at which the tangents are perpendicular to the initial line. Sketch the curve showing clearly the tangents at the pole. **(6 marks)**

   (b) Prove that

$$\frac{1+\cosh x+\cosh 2x}{\sinh x+\sinh 2x}=\coth x$$ **(3 marks)**

Hence evaluate $\displaystyle\int_1^e \frac{1+\cosh 2x+\cosh 4x}{\sinh 2x+\sinh 4x}\,dx$ **(3 marks)**

5. (a) Evaluate $\lim\limits_{x\to1}\dfrac{4x\ln x}{(x-1)}$ (4 marks)

(b) Given the curve $y = f(x)$, where

$$f(x) = \frac{ax^2}{\text{[illegible]} - b} \text{ and } f(x) \le 0 \text{ or } f(x) \ge 8$$

(i) Show that $x = 2$ and $y = x + 2$ are asymptotes to this curve. (3 marks)

(ii) Sketch the curve $y = f(x)$. (3 marks)

(iii) Find the centre of symmetry of the curve and show that the angle between the asymptotes is $\dfrac{\pi}{4}$. (2 marks)

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6. (a) A function f is defined by

$$f(x) = \ln(1 + e^x).$$

Derive the first three non-zero terms of the Maclaurin's series expansion of f. (3 marks)

Using the Maclaurin's series expansion for $\sin x$ and $\ln\left(1 - \frac{x}{2}\right)$, or otherwise, show that

$$\lim_{x\to0}\left(\frac{\ln\left(\frac{1+e^x}{2}\right) + \ln\left(1 - \frac{x}{2}\right)}{x - \sin x}\right) = -\frac{1}{4}$$ (2 marks)

(b) Verify that $k(k+1)$ is an even number $\forall k \in \mathbb{Z}$. (3 marks)

Hence prove that if a and b are odd integers $(a^2 - b^2)$ is a multiple of 8. (2 marks)

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7. Given that $I_n = \int_0^1 x^n\sqrt{(1-x)}\,dx$

Find I_0 and show that (2 marks)

$$I_n = \frac{2n}{2n+3}I_{n-1},\ n \ge 1.$$ (3 marks)

Hence, or otherwise, evaluate I_3. (3 marks)

Show also that

$$0 \le I_n \le \frac{1}{n+1} \text{ and evaluate } \lim_{n\to\infty} I_n.$$ (3 marks)

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8. (a) Show that the set $\{1, 3, 5, 7\}$ forms a group under multiplication modulo 8. *(Assume Associativity)* (5 marks)

(b) Solve the equation $X \otimes \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ where $\otimes$ is the composition of permutations. (3 marks)

9. Given that $\lambda x + t^2 y = 4t$ is a tangent to the rectangular hyperbola $xy = 4$ at the point $P\left(2t, \frac{2}{t}\right)$, find the real value(s) of λ. **(3 marks)**

This tangent meets the x-axis at A and the y-axis at B. The line through A parallel to the y-axis meets the hyperbola at C, while the line through B parallel to the x-axis meets the hyperbola at D. Show that as P varies, the locus of the midpoint of CD is the rectangular hyperbola $4xy = 25$. **(6 marks)**

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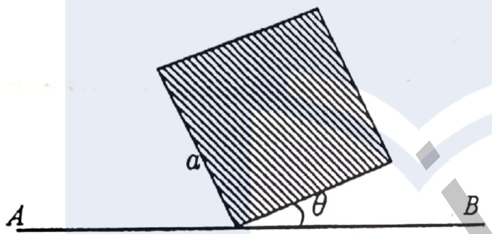
10. (a) The figure shows a square lamina of side a with its vertex inclined at an angle θ, $0 \le \theta \le \frac{\pi}{2}$, placed on a line AB,



Show that the centre of the lamina is at a distance

$$\frac{a\sqrt{2}}{2}\sin\left(\theta + \frac{\pi}{4}\right) \text{ from } AB.$$ **(3 marks)**

The lamina is rotated completely about the line AB, show that the volume of the solid generated is

$$\sqrt{2}\pi a^3 \sin\left(\theta + \frac{\pi}{4}\right).$$ **(3 marks)**

(b) Find the greatest common divisor (gcd), t, of 427 and 1037. Hence find integers r and s such that

$$427r + 1037s = t.$$ **(4 marks)**

Deduce that $7r + 17s = 1$.

END