GENERAL CERTIFICATE OF EDUCATION (GCE) BOARD

General Certificate of Education Examination

0765 Pure Maths with Mechanics 1

JUNE 2021 **ADVANCED LEVEL**

Centre Number	http://www.gcerevision.com
Centre Name	
Candidate Number	
Candidate Name	gcerevision.com

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTION PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed "**Advanced Level– 0765 Pure Maths with Mechanics 1".**
4. Fill in the information required in the spaces above.
5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil: **Candidate Number and Name,Centre Number and Name.** Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.
6. ***Answer All questions.***
7. **Formulae Booklets and calculators are allowed.**
8. Each question has FOUR suggested answers: **A, B, C** and **D**. Decide on which answer is correct. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square brackets for the answer you have chosen.
   For example, if **C** is your correct answer, mark **C** as shown below:
   [A] [B] [~~C~~] [D]
9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet, using, where necessary, the blank spaces in the question booklet.
12. **At the end of the examination, the invigilator shall collect the answer sheet first then the question booklet after. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

SECTION A: PURE MATHEMATICS

1. The vector equation of a line passing through the point (1, 3, 4) and parallel to the vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ is

A $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(-4\mathbf{i} - 2\mathbf{k})$
B $\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(-4\mathbf{i} - 2\mathbf{k})$
C $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$
D $\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$

2. A binary relation R is defined on $\mathbb{N}$, the set of natural numbers, by $m\,R\,n \Leftrightarrow m + n$ is odd. R is

A Symmetricand reflexive
B Symmetric only
C Symmetric and transitive
D Anti-symmetric

3. Given that the roots of the equation

$$x^2 - 14x + k = 0$$

are m and n and that $3m = 4n$, the value of the constant k is

A 28
B 32
C 24
D 48

4. The gradient of the tangent to the curve with parametric equations $x = \frac{2}{t}$ and $y = 3t$, where t is a parameter, at the point where $t = 1$ is

A $-\frac{2}{3}$
B $\frac{2}{3}$
C $\frac{-3}{2}$
D $\frac{3}{2}$

5. The range of values of x for which $|x| > 2 - |x|$ is

A $x < -1$ or $x > 1$
B $-2 < x < 2$
C $x < -2$ or $x > 2$
D $-1 < x < 1$

6. When a polynomial function $f(x)$ is divided by $x - 1$, the quotient is $2x^2 + x - 2$ and the remainder is 3. The polynomial $f(x) =$

A $2x^3 - x^2 - 3x + 2$
B $2x^3 - x^2 - 3x + 5$
C $2x^3 - x^2 - 3x + 3$
D $2x^3 - x^2 - x + 2$

7. The value of θ for which

$$\frac{2}{3\sin\left(\theta - \frac{\pi}{3}\right) + 5}$$

is minimum is

A $\frac{5\pi}{6}$
B $\frac{\pi}{6}$
C $-\frac{\pi}{2}$
D $\frac{\pi}{2}$

8. The set of values of x for which

$$(x - 1)(x - 3)(2x + 1) > 0$$

is

A $x < \frac{-1}{2}$ or $x > 3$
B $x < -3$ or $-1 < x < \frac{1}{2}$
C $x < \frac{1}{2}$ or $1 < x < 3$
D $\frac{-1}{2} < x < 1$ or $x > 3$

9. Given that

$y = x^{\sin x}$, where $x > 0$,

$\frac{dy}{dx} =$

A $(x\cos x)y$
B $\left(-\cos x \ln x + \frac{\sin x}{x}\right)y$
C $\left(\frac{\cos x}{x}\right)y$
D $\left(\cos x \ln x + \frac{\sin x}{x}\right)y$

10. Let $\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} m & n \\ p & q \\ r & s \end{pmatrix}$.

The order of the matrix product **AB** is

A 2×3
B 3×2
C 3×3
D 2×2

11. The number of arrangements of the letters of the word *MANNA* in which the *A*'s are together is

A 4
B 6
C 12
D 18

12. $f(x) = \begin{cases} x^2 + 2, & 0 \le x < 2 \\ 3x, & 2 \le x \le 4 \end{cases}$. Given that $f(x) = f(x + 4k)$, where $k \in \mathbb{Z}$, $f(15) =$

A 45
B 11
C 4
D 9

13. If the statements p and q are
p : John is eating,
q :John is playing,
the proposition $\sim q \rightarrow p$ is

A If John is eating then he is not playing
B If John is eating then he is playing
C If John does not play then he will not eat
D If John is not playing then he is eating

14. Values of y for various values of x are as shown in the table.

x	0	1	2	3	4
y	4	6	7	5	4

Using the trapezium rule $\int_0^4 y dx \approx$

A 9
B 22
C 44
D $\frac{44}{3}$

15. The variables x and y are related by the law
$y = a^2 b^x$.
Reducing this law to linear form gives

A $\log y = x \log b + 2 \log a$
B $\log y = x \log b + a \log 2$
C $\log y = 2 \log b + x \log a$
D $\log y = b \log x + 2 \log a$

16. The general solution of the differential equation
$\cos x \frac{dy}{dx} = y \sin x$
is

A $y = \sec^2 x + k$
B $y = \ln|\sec x| + k$
C $\ln y = \ln|\sec x| + k$
D $\ln y = \sec^2 x + k$

17. The sum S_n, of the first n terms of a sequence, is
$S_n = n(1 + 2n) \ln 2$.
The 5^{th} term of the sequence is

A $3 \ln 2$
B $19 \ln 2$
C $55 \ln 2$
D $5 \ln 2$

18. Given that the equation $4x^2 - 3kx + 1 = 0$ has equal roots and $k > 0$, the value of the constant k is equal to

A $\frac{3}{2}$
B $\frac{4}{3}$
C $\frac{2}{3}$
D $\frac{3}{4}$

19. The equation of the circle with centre at the point (1, 3) and passing through the point (2,7) is

A $(x-3)^2 + (y-1)^2 = 17$
B $(x-1)^2 + (y-3)^2 = 17$
C $(x-2)^2 + (y-7)^2 = 17$
D $(x-2)^2 + (y-3)^2 = 17$

20. Given that
$\frac{2x-3}{(x-3)(x-2)} \equiv \frac{P}{x-3} + \frac{Q}{x-2}$,
the values of P and Q are respectively

A -3 and 1
B -3 and -1
C 3 and -1
D 3 and 1

21. The range of values of x for which
$\frac{x+3}{x} > 0, \quad x \neq 0,$
is

A $-3 < x < 0$
B $0 < x < 3$
C $x < -3$ or $x > 0$
D $x < 0$ or $x > 3$

22. The equation of a circle with end-points $(4, -2)$ and $(3, 2)$ of its diameter is

A $(x-4)(x-3)+(y+2)(y-2)=0$
B $(x-4)(x+2)+(y-2)(y-2)=0$
C $(x-4)(x-3)+(y-2)(y-2)=0$
D $(x-4)(x-2)+(y+2)(y-2)=0$

23. $\int_{-1}^{2} \frac{3+x}{2+x} dx =$

A $\frac{3}{2}$
B $\frac{1}{4}+\ln 4$
C 1
D $3+\ln 4$

24. The expansion of the function $\frac{1}{(1-3x)(1+x)}$ is valid for

A $-\frac{1}{3} \leq x \leq \frac{1}{3}$
B $-1 < x < 1$
C $-\frac{1}{3} < x < \frac{1}{3}$
D $-3 < x \leq 3$

25. The Cartesian equation of the curve whose parametric equations are $x-1=\sec\theta$ and $y+1=\tan\theta$ is

A $y^2-x^2+1=0$
B $x^2-y^2-2x-2y-1=0$
C $y^2+3x^2+1=0$
D $x^2+y^2+2x+2y+1=0$

26. If the matrix $\begin{pmatrix} 6 & k & 2 \\ 4 & -3 & 3 \\ 2 & -k & 2 \end{pmatrix}$ is not invertible (i.e is singular), the value of k is

A -2
B -3
C 2
D 3

27. $\int 2x(x^2+3)^{\frac{3}{2}} dx =$

A $\frac{1}{5}(x^2+3)^{\frac{5}{2}}+k$
B $\frac{2}{5}\left(\frac{x^4}{4}+\frac{3x^2}{2}\right)^{\frac{5}{2}}+k$
C $\frac{2}{5}(x^2+3)^{\frac{5}{2}}+k$
D $\frac{5}{2}(x^2+3)^{\frac{5}{2}}+k$

28. A first approximation to the root of the equation $e^x+2x-1=0$ is $x=1$. Using the Newton-Raphson method, a second approximation to the root of the equation is

A $1-\left(\frac{e+1}{e+2}\right)$
B $1-\left(\frac{e+2}{e+1}\right)$
C $1-\left(\frac{e-1}{e+2}\right)$
D $1+\left(\frac{e+1}{e+2}\right)$

29. The equation $x^3+3x-5=0$ has a root lying in the open interval

A $(2, 3)$
B $(0, 1)$
C $(1, 2)$
D $(-1, 0)$

30. Given that $|z-3|=2|z+3|$, where $z=x+iy$, the locus described by z is

A a circle
B a line parallel to the y-axis
C the y-axis
D a line parallel to the x-axis

31. $e^{x\ln 5} =$

A 5^x
B $\ln 5$
C e^{x^5}
D e^5

32. The approximate change in the value of $\ln x$ if x changes from 10 to 10.1 is

A 0.1
B 0.0001
C 0.001
D 0.01

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33. The vectors $2\mathbf{i} - q\mathbf{j} - \mathbf{k}$ and $-3\mathbf{i} - 2\mathbf{j} + q\mathbf{k}$ are perpendicular if the scalar q is equal to

A 1
B -3
C 6
D -2

34. The vector perpendicular to the plane $3x - 5y + z + 7 = 0$ is

A $3\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$
B $3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$
C $3\mathbf{i} + \mathbf{j} + 7\mathbf{k}$
D $3\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$

35. If $A = \tan^{-1}5 + \tan^{-1}(-3)$, then $\tan A =$

A $\frac{1}{2}$
B $\frac{-4}{7}$
C $\frac{1}{8}$
D $\frac{-1}{7}$

SECTION B: MECHANICS

36. The position vector of a particle is $\mathbf{r} = [(t^3 + 2t^2 + 2)\mathbf{i} + (t^2 - 1)\mathbf{j}]$ m at time t s. The velocity of the particle when $t = 1$ is

A $(9\mathbf{i} + \mathbf{j})$m s^{-1}
B $5\mathbf{i}$m s^{-1}
C $(10\mathbf{i} + 2\mathbf{j})$ m s^{-1}
D $(7\mathbf{i} + 2\mathbf{j})$ m s^{-1}

37. When the length of a spring is 60% of its natural length, the thrust in the spring is 10 N. the modulus of the spring is

A $\frac{50}{3}$ N
B $\frac{20}{3}$ N
C 25 N
D 4 N

38. A centripetal force of 72 N causes a particle of mass 4 kg to move in a horizontal circle of radius 2 m. The angular speed of the particle is

A 9 rad s^{-1}
B 3 rad s^{-1}
C 9 m s^{-1}
D 3 m s^{-1}

39. Fig. 1 below shows four particles placed at the vertices of rectangle $ABCD$.

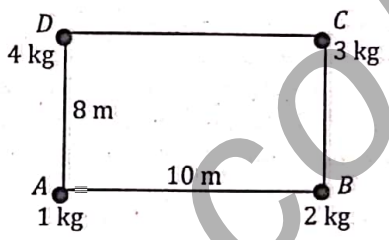

Fig. 1

The distance of the center of gravity from the side AB is

A 4 m
B 5.6 m
C 11 m
D 5.5 m

40. A particle of mass 15 kg rests on a smooth horizontal table. It is connected by a light inextensible string passing over a smooth pulley fixed at the edge of the table to a particle of mass 10 kg which hangs freely. Given that the acceleration of the system is 4 m s^{-2} when it is released from rest, the force exerted by the string on the pulley is

A $120\sqrt{2}$ N
B $60\sqrt{2}$ N
C 60 N
D 120 N

41.

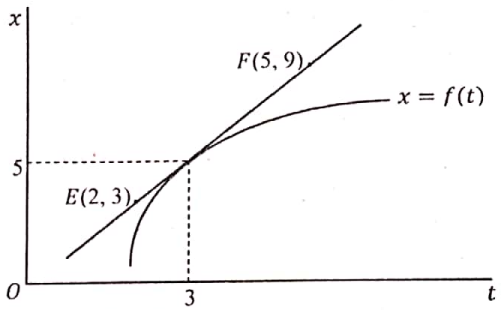

Fig. 2

Fig. 2 shows a sketch of the distance-time graph of a particle whose distance x m from a fixed point at time t s is given by $x = f(t)$. The tangent to the curve at $t = 3$ is shown passing through two distinct points E and F. The speed of the particle when $t = 3$ is

A $4\ \text{m s}^{-1}$
B $2\ \text{m s}^{-1}$
C $3\ \text{m s}^{-1}$
D $5\ \text{m s}^{-1}$

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42. A uniform ladder AB, of length $2a$, rests in limiting equilibrium with its top end against a rough vertical wall and its lower end on a rough horizontal floor. The forces acting on the ladder are as shown in Fig. 3 below.

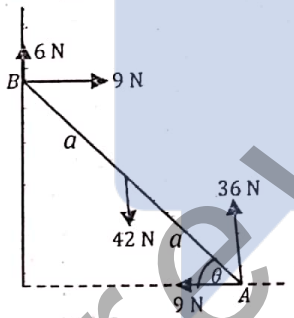

Fig. 3

The angle θ which AB makes with the floor is given by

A $\tan\theta = \frac{3}{5}$
B $\tan\theta = \frac{5}{3}$
C $\sin\theta = \frac{3}{5}$
D $\sin\theta = \frac{5}{3}$

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43. The velocity of X is $(4\mathbf{i} + 5\mathbf{j})\ \text{m s}^{-1}$. Relative to X the velocity of Y is $(3\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$ and relative to Y the velocity of Z is $(5\mathbf{i} - 3\mathbf{j})\ \text{m s}^{-1}$. The true velocity of Z is

A $(12\mathbf{i} + 4\mathbf{j})\ \text{m s}^{-1}$
B $(9\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$
C $(7\mathbf{i} + 7\mathbf{j})\ \text{m s}^{-1}$
D $(8\mathbf{i} - \mathbf{j})\ \text{m s}^{-1}$

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44. A particle accelerating from rest has its speed $v\ \text{m s}^{-1}$ at time t s given by

$$v = 6t - \frac{1}{2}t^2.$$

The acceleration of the particle when $v = 10$ is

A $1\ \text{m s}^{-2}$
B $10\ \text{m s}^{-2}$
C $2\ \text{m s}^{-2}$
D $4\ \text{m s}^{-2}$

---

45. A smooth sphere A of mass 4 kg travelling at $5\ \text{m s}^{-1}$ collides directly with another smooth sphere B of mass 3 kg travelling at $4\ \text{m s}^{-1}$ in the opposite direction. Given that the speed of A after impact is $0.5\ \text{m s}^{-1}$, the kinetic energy of B after impact is

A 24 J
B 3 J
C 6 J
D 6.5 J

---

46. The work done by a force $\mathbf{F} = (\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ N which moves its point of application from a point A with position vector $\mathbf{r}_A = (3\mathbf{i} - 4\mathbf{k})$ m to another point B with position vector $\mathbf{r}_B = (-\mathbf{i} + 6\mathbf{k})$ m is

A −24 J
B −44 J
C 24 J
D 44 J

---

47. The engine of a car of mass 1,200 kg works at a constant rate of 54 kW up a road inclined at an angle $\sin^{-1}\left(\frac{1}{5}\right)$ to the horizontal. Given that the non-gravitational resistance to the motion of the car is 300 N and taking g as 10 m s^{-2}, the maximum speed of the car is

A $20\ m\ s^{-1}$
B $25\ m\ s^{-1}$
C $100\ m\ s^{-1}$
D $900\ m\ s^{-1}$

48. A coin is biased in such a way that the probability of it landing head-up is $\frac{3}{4}$. The coin is tossed 3 times, the probability of getting exactly one tail is

A $\frac{9}{64}$
B $\frac{1}{64}$
C $\frac{27}{64}$
D $\frac{55}{64}$

49. Two particles of masses 2 kg and 3 kg are placed the points (3,1) and (8, 6) respectively. The position of their center of mass is at the point

A (6, 4)
B (5·5, 3·5)
C (5, 3)
D (4, 6)

50. A force of 14 N acts on a particle of mass 70 kg causing the speed of the particle to increase from 3 m s^{-1} to 7 m s^{-1}. The distance travelled by the particle during this period is

A 40 m
B 4 m
C 0·3 m
D 100 m

NOW GO BACK AND CHECK YOUR WORK