GENERAL CERTIFICATE OF EDUCATION (GCE) BOARD
General Certificate of Education Examinations

0765 Pure Maths With Mechs 3

JUNE 2021 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics with Mechanics**
Paper No.	**Paper 3**
Subject Code No.	**0765**

Three hours

Full marks may be obtained for answers to ALL questions.

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae booklet published by Board are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. A particle P of unit mass is moving on a horizontal plane under the action of a force **F** at time t seconds,where

$$\mathbf{F} = [(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}] \text{ N}$$

Given that P passes through a point A with velocity $(11\mathbf{i} + 10\mathbf{j})$ m s^{-1} after 3 seconds,

find

(a) the acceleration of P at time t seconds, **(2 marks)**

(b) the velocity of P at time t seconds, **(4 marks)**

(c) the kinetic energy of P when it is moving parallel to the vector **i**. **(5 marks)**

Another particle Q moves in the same straight path as P with velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}.

(d) Determine the velocity of P relative to Q at time $t = 2$. **(2 marks)**

2.

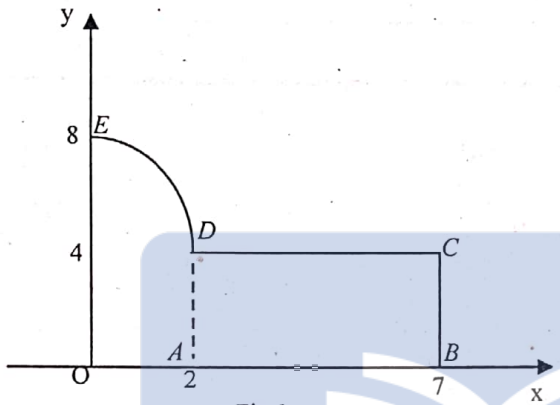

Fig.1

In Fig.1 above ED is a portion of the curve $y = 8 - x^2$ and $ABCD$ is a rectangle in which $BC = 4$ cm and $CD = 5$cm.

(a) Show , by integration, that the coordinates of the centroid of the portion $OADE$ is $\left(\frac{9}{10}, \frac{86}{5}\right)$. **(6 marks)**

(b) Find the distance of the centre of mass of the whole lamina OBCDE from OE. **(7 marks)**

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3. A uniform rod AB of mass $2m$ kg and length $4a$ metres is smoothly hinged at a point on a vertical wall.It is supported in a horizontal position by a light inextensible string attached to the rod at point C, where $CB = a$. The other end of the string is attached to the point D on a vertical wall with D above A such that the string makes an angle 60° with the wall. A load of mass m kg is placed at B and the system is in equilibrium.

(a) Show that the tension in the string is $\frac{16}{3}mg$. **(7 marks)**

(b) Find , in terms of m and g, the horizontal and the vertical components of the reaction force at the hinge. **(4 marks)**

(c) Determine, to one decimal place, the angle which the reaction force makes with the rod. **(2 marks)**

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4. (i) A particle is projected from a point O with initial speed 30 m s^{-1} and passes through a point which is at a horizontal distance 60 m from O and a distance 20 m vertically above the level of O.

(a) Show that there are two possible angles of projection **(4 marks)**

(b) If these angles are α and β, prove that $\tan(\alpha + \beta) = -3$ **(3 marks)**

(ii) A particle of mass m kg moves on the inside surface of a smooth spherical bowl of radius $2a$, describing a horizontal circle at a distance a below the centre of the bowl.

(c) Prove that its speed is $\sqrt{3ag}$. **(4 marks)**

(d) Find the period of revolution. **(2 marks)**

5.

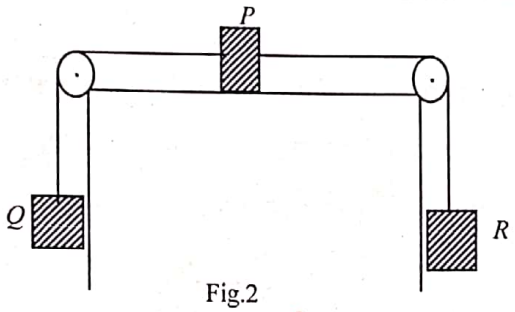

Fig.2

Fig.2 above shows a block P of mass 2 kg which lies on a rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{2}$. P is connected by two light inextensible strings passing over two fixed smooth pulleys to blocks Q and R of masses m kg and 7 kg respectively. The blocks hang freely and the system is released from rest with both strings taut and the hanging parts vertical. Given that the tension in the string PQ is 37.5 N,

find

(a) the acceleration of the system and the tension in the string PR. **(7 marks)**

(b) the value of m. **(2 marks)**

R strikes the horizontal ground 2 second after release and does not rebound. Assuming that in their subsequent motion P and Q do not reach the pulleys.

(c) Calculate the new acceleration of the system and the new tension in the string PQ. **(4 marks)**

(Take g as 10 m s^{-2}.)

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6. Two smooth spheres A and B of equal radii and masses m and $2m$ respectively lie at rest on a smooth horizontal table. Sphere A is projected towards sphere B with speed u so that they collide directly. Given that the coefficient of restitution between A and B is $\frac{1}{3}$,

find

(a) the speed of each sphere after impact, **(7 marks)**

(b) the magnitude of the impulse exerted by A on B, **(2 marks)**

(c) the loss of kinetic energy due to the impact. **(4 marks)**

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7. The engine of a car works at constant rate of 15 kW. The car travels along a straight level road with maximum speed of 120 km h^{-1}.

(a) Calculate the total resistance to the motion of the car. **(3 marks)**

Given that the mass of the car is 1000 kg and that the resistance to motion is proportional to the square of the speed, find

(b) the rate at which the engine is working when the car is moving at constant speed of 40 km up a road of inclination θ, where $\sin\theta = \frac{1}{25}$. **(6 marks)**

If the engine of the car is shut off at the instant when the speed is 40 km h^{-1}, find

(c) the time the car takes to come to momentary rest. **(4 marks)**

8 (i) Two independent events A and B are such that $(A) = \frac{2}{5}$, $P(A|B) = \frac{1}{3}$ and $P(B|A) = \frac{1}{2}$.
Find

(a) $P(B)$, **(3 marks)**

(b) $P(A \cup B)$. **(3 marks)**

(ii) A spare parts dealer receives spare parts from three different suppliers X, Y and Z, with $\frac{2}{5}$ of the parts coming from X, $\frac{7}{20}$ from Y and the rest from Z. It is known that $\frac{2}{25}$ of the parts supplied by X, $\frac{1}{10}$ supplied by Y and $\frac{1}{20}$ supplied by Z are defectives. If a spare part is bought from the dealer,

find the probability that the spare part is

(c) defective, **(3 marks)**

(d) either from Y or it is defective, **(2 marks)**

(e) from X given that it is not defective. **(2 marks)**