

GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination.

0775 Further Mathematics 1

JUNE 2022

ADVANCED LEVEL

Centre Number	
Centre Name	
Candidate Number	
Candidate Name	

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTION PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper.

Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed "Advanced Level – 0775 Further Mathematics 1".
4. Fill in the information required in the spaces above.
5. Fill the information required in the spaces provided on the answer sheet using your HB pencil: Candidate Name, Exam Session, Subject Code and Candidate Identification Number.

Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.

How to answer the questions in this examination

6. Answer ALL the 50 questions in this Examination. All questions carry equal marks.
7. Calculators are allowed.
8. Each question has FOUR suggested answers: A, B, C and D. Decide on which answer is correct. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square bracket for the answer you have chosen.

For example, if C is your correct answer, mark C as shown below:

[A] [B] ☒ [C] [D]

9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet, using the blank spaces in the question booklet.
12. At the end of the examination, the invigilator shall collect the answer sheet first and then the question booklet. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.

Turn Over

1. The complementary function of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 10x + 1, \text{ where } P, Q \in \mathbb{R} \text{ is}$$

- A $y = P + Qe^{3x}$
- B $y = Pe^{-3x} + Q$
- C $y = (P + Qx)e^{-3x}$
- D $y = P \cos 3x - Q \sin 3x$

2. $\frac{(x+4)}{(x^2-1)(x+1)}, x \neq \pm 1$, can be expressed in partial fractions, where P, Q , and R are constants as

- A $\frac{P}{x-1} + \frac{Q}{x+1} + \frac{R^2}{(x+1)^2}$
- B $\frac{P_r + Q}{x^2-1} + \frac{R}{x+1}$
- C $\frac{P}{x-1} + \frac{Q}{(x-1)^2} + \frac{R}{x+1}$
- D $\frac{P}{(x-1)} + \frac{Q}{x+1} + \frac{Rx}{(x+1)^2}$

3. The function $f(x) = \frac{(x-3)(x+1)}{x(x-3)(x+3)}$ has a removable discontinuity at $x =$

- A 0
- B 1
- C 2
- D 3

4. $\lim_{x \rightarrow \infty} \frac{\cos x}{x} =$

- A ∞
- B 0
- C 1
- D $-\infty$

5. Which one of the polar curves is symmetrical about the line $\theta = 0$ only

- A $r = 1 + \cos \theta$
- B $r = 1 + \sin \theta$
- C $r = \sin 2\theta$
- D $r = \cos 2\theta$

6. If the motion modelled by $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + n^2x = 0$ is a damped harmonic motion then

- A $k < n$
- B $k \leq n$
- C $0 < k < n$
- D $0 \leq k \leq n$

7. Given that $\mathbf{p}$ and $\mathbf{q}$ are two non-zero vectors such that $\mathbf{p} - \alpha\mathbf{q} = \mathbf{0}$, where α is a real constant. Which one of the following is true?

- A $\mathbf{p} \cdot \mathbf{q} = 0$.
- B $\mathbf{p} \cdot \mathbf{q} = \alpha$
- C $\mathbf{p} \times \mathbf{q} = 0$.
- D $|\mathbf{p} \times \mathbf{q}| = \alpha$.

8. The structure $(\mathbb{N}, +)$ is **NOT** a group because $(\mathbb{N}, +)$

- A is not closed
- B is associative
- C does not have an identity element
- D does not have inverse elements

9. If $2x \equiv 3 \pmod{7}$, then $x =$

- A 2
- B 3
- C 4
- D 5

10. Given that $\frac{dy}{dx} = x - \frac{y}{x}$ and $y = 0$ when $x = 1$, then using the approximation

$y_{n+1} \cong y_n + hy'_n$, $h = 0.1$, the value of y when $x = 1.1$ is

- A 0.1
- B 0.01
- C 0
- D 0.2

11. A, B, C are the position vectors of the vertices of a triangle. The area of triangle ABC is

- A $\left| \frac{1}{2} \mathbf{AB} \times \mathbf{AC} \right|$
- B $\left| \frac{1}{2} \mathbf{AB} \times \mathbf{BC} \right|$
- C $\left| \frac{1}{2} \mathbf{AC} \times \mathbf{BC} \right|$
- D $\left| \frac{1}{2} \mathbf{AB} \times \mathbf{CB} \right|$

12. Given two statements

$$p: \sinh x = \sinh y$$

$$q: x = y$$

The contrapositive of $p \Rightarrow q$ is

- A If $x \neq y$, then $\sinh x \neq \sinh y$
- B If $x = y$, then $\sinh x \neq \sinh y$
- C If $x = y$, then $\sinh x = \sinh y$
- D If $x \neq y$, then $\sinh x = \sinh y$

13. If $\sinh x = \frac{1}{2}$ then $\cosh 2x =$

- A $\frac{1}{2}$
- B $\frac{3}{2}$
- C 1
- D $\frac{\sqrt{3}}{2}$

14. Given that $|z| = 3$, the maximum value of $|z - 2i|$ is

- A 1
- B 2
- C 3
- D 5

15. The Taylor series expansion of the function

$$f(x) = \ln(3 - 4x)$$

- A $]-\frac{3}{4}, \frac{3}{4}]$
- B $[-\frac{3}{4}, \frac{3}{4}]$
- C $]-\frac{3}{4}, \frac{3}{4}[$
- D $[-\frac{3}{4}, \frac{3}{4}[$

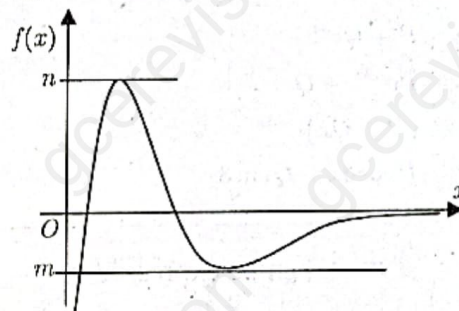
16. If $u_1 = \frac{11}{2}$, $u_{n+1} = \lambda u_n$, $n \geq 1$, then u_n can be expressed as

- A $(\frac{11}{2})\lambda^n$
- B $(\frac{11}{2})^n \lambda$
- C $(\frac{11}{2})\lambda^{n-1}$
- D $(\frac{11}{2})^{n-1} \lambda$

17. For $x > 1$, $\int \frac{1}{x \ln x} dx =$

- A $\ln x + k$
- B $\ln(\ln x) + k$
- C $\ln^2 x + k$
- D $2 \ln x + k$

18.



From the graph of f , the equation $f(x) = a$, $a \in \mathbb{R}$, has only one root for

- A $a > n$
- B $a > m$
- C $n < a < m$
- D $a < m$

19. The root mean square value of $\sinh x$, for $1 \leq x \leq 3$ is

- A $\left[\frac{1}{2} \int \sinh^2 x dx \right]^{\frac{1}{2}}$
- B $\frac{1}{2} \left[\int_1^3 \sinh^2 x dx \right]^{\frac{1}{2}}$
- C $\left[\frac{1}{2} \int_1^3 \sinh^2 x dx \right]^{\frac{1}{2}}$
- D $\frac{1}{3} \left[\int_1^3 \sinh^2 x dx \right]^{\frac{1}{2}}$

20. $P(X = x) = e^{-kx}$, $x \geq 0$. The mean of X is

- A k^2
- B $\frac{1}{k^2}$
- C $-k^2$
- D $-\frac{1}{k^2}$

21. The domain of the function $g(x) = \frac{x+1}{\sqrt{x^2-9}}$ is

- A $\{x \in \mathbb{R} : x < -3\} \cup \{x \in \mathbb{R} : x > 3\}$
- B $\{x \in \mathbb{R} : -3 \leq x \leq 3\}$
- C $\{x \in \mathbb{R} : x \leq -3\} \cup \{x \in \mathbb{R} : x \geq 3\}$
- D $\{x \in \mathbb{R} : -3 \leq x < -1\} \cup \{x \in \mathbb{R} : -1 < x \leq 3\}$

Turn Over

22. Given the sequence (u_n) where

$$u_0 = 3, u_{n+1} = \frac{2}{3}u_n, \text{ then } \lim_{n \rightarrow \infty} u_n =$$

- A 0
- B 1
- C $\frac{2}{3}$
- D 3

23. The image of the line $y = 2x$ under the

transformation matrix $\begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$ is the line

- A $y = x$
- B $y = 2x$
- C $y = -4x$
- D $y = 1 - x$

24. $\int_0^1 \frac{1}{9 - x^2} dx =$

- A $\sinh^{-1}\left(\frac{1}{3}\right)$
- B $\cosh^{-1}\left(\frac{1}{3}\right)$
- C $\tanh^{-1}\left(\frac{1}{3}\right)$
- D $\cosh^{-1}(3)$

25. The work done when the point of application of the force $\mathbf{F}$ is displaced from $\mathbf{r}_2$ to $\mathbf{r}_1$ is

- A $(\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F}$
- B $(\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{F}$
- C $(\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$
- D $(\mathbf{r}_1 - \mathbf{r}_2) \cdot \mathbf{F}$

26. A smooth sphere is moving with velocity $(-3\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$ when it hits a smooth wall parallel to the vector $\mathbf{j}$. If the coefficient of restitution between the sphere and the wall is $\frac{1}{3}$, then the velocity of the sphere after hitting the wall is

- A $\mathbf{i} + \mathbf{j}$
- B $\mathbf{i} - \mathbf{j}$
- C $-\mathbf{i} + \mathbf{j}$
- D $-\mathbf{i} - \mathbf{j}$

27. Given that $z, w \in \mathbb{C}$, then the transformation $w = -z$ is

- A a rotation $\frac{\pi}{2}$ clockwise
- B a rotation $\frac{\pi}{2}$ anticlockwise
- C a rotation π clockwise
- D a rotation $\frac{-\pi}{2}$ anticlockwise

28. If $25 \cosh x + 7 \sinh x \equiv R \sinh(x + \lambda)$, then the value of R is

- A $\sqrt{674}$
- B 24
- C 18
- D 16

29. The moment of inertia of a rigid body of mass $2M$ about a given axis is $\frac{Ma^2}{8}$. The radius of gyration about the axis is

- A $\frac{a}{\sqrt{8}}$
- B $\frac{a}{4}$
- C $\frac{a}{2}$
- D $\frac{a^2}{4}$

30. The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{k}{2}(x^2 - 1), & -1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

The value of k is

- A $-\frac{8}{3}$
- B -2
- C $-\frac{3}{8}$
- D $-\frac{4}{3}$

31. The number, n , of bees in a large apiary grows at a rate directly proportional to the number of bees present at time t . The differential equation that models the number of bees in this apiary, for $k > 0$, is

- A $\frac{dn}{dt} = kn$
- B $\frac{dn}{dt} = -k$
- C $\frac{dn}{dt} = k$
- D $\frac{dn}{dt} = -kn$

32. The asymptotes to the hyperbola $9x^2 - 4y^2 = 36$ are $y =$

- A $\pm \frac{9}{4}x$
- B $\pm \frac{4}{9}x$
- C $\pm \frac{2}{3}x$
- D $\pm \frac{3}{2}x$

33. If $5x \equiv 2 \pmod{16}$ then one value of x is

- A 0
- B 4
- C 6
- D 10

34. The integrating factor of the differential

equation $\frac{dy}{dx} + y \tanh x = \cos x$ is

- A $\cosh x$
- B $\operatorname{sech} x$
- C $e^{\cosh x}$
- D $e^{\operatorname{sech} x}$

35. The negation of the statement $P \wedge \sim Q$ is

- A $P \vee Q$
- B $\sim P \wedge Q$
- C $P \wedge Q$
- D $\sim P \vee Q$

36. The work done against a force $\mathbf{F}$ in moving its point of action from point $\mathbf{a}$ to point $\mathbf{b}$ is

- A $(\mathbf{b} - \mathbf{a}) \cdot \mathbf{F}$
- B $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{F}$
- C $(\mathbf{b} - \mathbf{a}) \times \mathbf{F}$
- D $(\mathbf{a} - \mathbf{b}) \times \mathbf{F}$

37. Given that $h\left(\frac{dy}{dx}\right)_n \approx y_{n+1} - 2y_n$ and $\frac{dy}{dx} = x + 2$,

then $y_{n+1} \approx$

- A $2y_n + hx_n + 2$
- B $2y_n + h(x_n + 2)$
- C $2y_n + h(x_n - 2)$
- D $2y_n - h(x_n + 2)$

38. The function $f(x) = \frac{(x^2 + x)(x + 2)(x + 3)}{x(x - 1)(x - 2)}$ has a removable discontinuity at $x =$

- A 0
- B 1
- C 2
- D 3

39. A particle moves along the curve $r = 1 - \cos \theta$ with constant angular velocity w . The maximum speed occurs for $\theta =$

- A $\frac{\pi}{2}$
- B π
- C $\frac{3\pi}{2}$
- D 2π

40. Which one of the following lines is invariant under

the transformation with matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$?

- A $\frac{1}{2}x = y = \frac{1}{3}z$
- B $2x = y = 3z$
- C $\frac{1}{2}x = y = z$
- D $x = \frac{y}{2} = \frac{z}{3}$

41. For what value(s) of n is $\int_0^2 x^n dx = \int_0^2 (2 - x)^n dx$

- A $n = 2k, k \in \mathbb{Z}$
- B $n = 2k + 1, k \in \mathbb{Z}$
- C $n \in \mathbb{R}$
- D $n \notin \mathbb{R}$

42. $z = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$, then $\left|z + \frac{1}{z}\right| =$

- A $\frac{\sqrt{3}}{2}$
- B 1
- C $\sqrt{3}$
- D $2\sqrt{3}$

43. A binary operation $\bullet$ is defined on $\mathbb{R}$, as

$$x \bullet y = x + y + 1. \text{ The inverse of 2 is}$$

- A -4
- B -2
- C 0
- D 4

Turn Over

44. The first three terms in the Maclaurin expansion of e^{3x} are

- A $1 + 3x + \frac{9}{2}x^2$
- B $1 + 3x + 9x^2$
- C $1 + 3x + \frac{9}{4}x^2$
- D $1 + 3x + 3x^2$

45. A sequence (u_n) is defined recursively by

$$\begin{cases} u_0 = 3, \\ u_{n+1} = \frac{2}{3}u_n. \end{cases}$$

$u_n =$

- A $2^n 3^{1-n}$
- B $2^n 3^n$
- C $2^{1-n} 3^n$
- D $2^n 3^{n-1}$

46. The asymptote to the curve $y + 1 = 2^x$ is

- A $x = 0$
- B $y = -1$
- C $y = 1$
- D $x = 1$

47. The arc length of the curve $f(x) = \cosh x$ for

$$0 \leq x \leq \ln 2 \text{ is}$$

- A $\frac{3}{4}$
- B $\frac{5}{4}$
- C $\frac{5}{2}$
- D $\frac{3}{2}$

48. For any three non-singular matrices A , B and C ,

$$A^{-1}(BC)^{-1} =$$

- A $A^{-1}C^{-1}B^{-1}$
- B $C^{-1}B^{-1}A^{-1}$
- C $A^{-1}B^{-1}C^{-1}$
- D $B^{-1}A^{-1}C^{-1}$

49. A force $\mathbf{F} = (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})\text{N}$ acts through the point $A(2, 2, 2)$. The moment of $\mathbf{F}$ about the point $B(0, 3, 1)$ is

- A $3\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$
- B $-3\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$
- C $-3\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}$
- D $-3\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$

50. Which one of the following functions is NOT differentiable in the interval $0 \leq x \leq 2$

- A $y = |x - 1|$
- B $y = x^2$
- C $y = -x^2$
- D $y = |x - 2|$

GO BACK AND CHECK YOUR WORK