

# GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

0775 FURTHER MATHS 3

JUNE 2022

ADVANCED LEVEL

Subject Title	Further Mathematics
Paper No.	Paper 3
Subject Code No.	0775

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**Two and a half hours**

**Answer ALL questions.**

*For your guidance the approximate mark allocation for parts of each question is indicated in brackets.*

*Mathematical formulae and tables, published by the Board, and noiseless non-programmable electronic calculators are allowed.*

*In calculations, you are advised to show all the steps in your working, giving your answer at each stage.*

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1. A rigid body is acted on by three forces  $F_1$ ,  $F_2$  and  $F_3$  where

$$F_1 = (14i + 8j + 8k)\text{N}, F_2 = (9i + 4j - 12k)\text{N}, F_3 = (24i + 8j - 9k)\text{N}.$$

The force  $F_1$  acts through the point with position vector  $(-i + 2j + k)\text{m}$ .

The forces  $F_2$  and  $F_3$  act through the point with position vector  $(i - j + k)\text{m}$ .

The system is equivalent to a single force  $F$  acting at the point  $(1, 1, 0)$  together with a couple  $G$ .

Find

- (i) the force  $F$  (2 marks)  
 (ii) the couple  $G$  (7 marks)  
 (iii) a Cartesian equation of the line of action of  $F$ . (2 marks)

2. A particle  $P$ , of mass  $m$ , moves along  $Ox$  under the action of a force of magnitude  $4m\omega^2|x|$  directed towards  $O$ , where  $\omega$  is a positive constant. The resistance to motion is of magnitude  $2\lambda m\omega|v|$ , where  $\lambda$  is a constant and  $v$  is the velocity of  $P$ .

- (a) Show that

$$\ddot{x} + 2\lambda\omega\dot{x} + 4\omega^2x = 0. \quad (4 \text{ marks})$$

- (b) Discuss the motion of  $P$  when

(i)  $\lambda = 0$  (5 marks)

(ii)  $\lambda = 1$  (2 marks)

(iii)  $\lambda = \frac{5}{2}$ . (2 marks)

3. The function  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = f(x, y) \text{ where } f(x, y) = 2 + \frac{y}{x} \text{ and } y(1) = 1.$$

- (i) Use the formula

$$y_{r+1} \cong y_r + hf(x_r, y_r) \text{ with } h = 0.05$$

to obtain an approximate value for  $y(1.1)$ , giving your answer correct to three decimal places. (4 marks)

- (ii) Use Simpson's rule to estimate the value of  $\int_1^{1.1} y dx$ . (4 marks)

4. A smooth sphere  $S$  of mass  $2kg$  is moving with speed  $u$  along a smooth horizontal floor when it collides obliquely with a smooth fixed vertical wall. Just before impact the direction of motion of  $S$  makes an angle  $\theta$  with the plane of the wall. After impact  $S$  moves with speed  $v$  in a direction making an angle of  $60^\circ$  with the plane of the wall. The coefficient of restitution between the sphere and the wall is  $\frac{1}{3}$ .

Show that

(i)  $\tan \theta = 3\sqrt{3}$  (7 marks)

(ii)  $v = \frac{u\sqrt{7}}{7}$  (4 marks)

Find the magnitude of the impulse received by the sphere during the impact. (2 marks)

5. A particle  $P$  moves with constant angular velocity  $\omega$  on the polar curve with equation  $r = f(\theta)$ . At time  $t$ , the transverse component of the acceleration of  $P$  is  $2a\omega^2 \cos \theta$ , where  $a$  is a positive constant. Initially  $P$  is at the point with polar coordinates  $(a, \frac{\pi}{2})$ .

(i) Show that the radial component of the velocity of  $P$  is  $a\omega \cos \theta$ . (4 marks)

(ii) Show also that

$$\frac{dr}{d\theta} = a \cos \theta. \quad (4 \text{ marks})$$

(iii) Find the polar equation of the path of  $P$ . (4 marks)

(iv) Find, in terms of  $\theta$ ,  $\omega$ , the radial acceleration of  $P$ . (2 marks)

6. (i) Show, by integration, that the moment of inertia of a thin uniform circular disc of mass  $2m$  and radius  $a$  about an axis through the centre,  $O$ , of the disc and perpendicular to the plane of the disc is  $ma^2$ . (4 marks)

The disc is free to rotate in a vertical plane about a smooth horizontal axis through a point  $A$  on the circumference of the disc. The axis is perpendicular to the plane of the disc.

(ii) Show that the moment of inertia of the disc about this axis is  $3ma^2$ . (2 marks)

The disc is released from rest when  $AO$  is horizontal.

(iii) Show that

$$3a \left( \frac{d\theta}{dt} \right)^2 = 4g \sin \theta, \text{ where } \theta \text{ is the angle } AO \text{ makes with the horizontal through } A. \quad (2 \text{ marks})$$

(iv) Find the component of the reaction at the axis perpendicular to  $AO$  when  $\theta = \frac{\pi}{4}$ . (6 marks)

7. A particle  $P$  of mass  $m$  falls from rest in a medium which offers a resistance to motion of magnitude  $\frac{mgv^2}{U^2}$  N where  $v$  in  $\text{m s}^{-1}$  is the speed and  $U$  is a positive constant.

(i) Show that when the speed of  $P$  is  $\frac{3}{5}U$ ,  $P$  has fallen a distance  $\frac{U^2}{g} \ln \left( \frac{5}{4} \right)$ . (7 marks)

(ii) Find the time taken by  $P$  to attain the speed of  $\frac{3}{5}U$ . (6 marks)

Turn Over

8 (a) A discrete random variable  $X$  has probability mass function defined by

$$f(x) = \begin{cases} c(9 - x^2), & \text{for } x = 1, 2 \\ c(25 - x^2), & \text{for } x = 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

Find

- (i) the value of the constant  $c$ , (2 marks)
  - (ii) the median of  $X$ , (2 marks)
  - (iii) the mean and standard deviation of  $X$ . (5 marks)
- (b) A continuous random variable  $Y$  has normal distribution with mean 9 and variance 4. Given that  $P(Y > k) = 0.025$ , determine the value of  $k$ . (5 marks)

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**GO BACK AND CHECK YOUR WORK**