GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

0765 Pure Maths with Mechanics 1

JUNE 2022 **ADVANCED LEVEL**

Centre Number	
Centre Name	
Candidate Number	
Candidate Name	

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTION PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed **"Advanced Level– 0765 Pure Maths with Mechanics 1".**
4. Fill in the information required in the spaces above.
5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil: **Candidate Number and Name, Centre Number and Name.**
   Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.
6. ***Answer All questions.***
7. **Formulae Booklets and calculators are allowed.**
8. Each question has FOUR suggested answers: **A, B, C** and **D**. Decide on which answer is correct. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square brackets for the answer you have chosen.
   For example, if **C** is your correct answer, mark **C** as shown below:
   [A] [B] [~~C~~] [D]
9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet, using, where necessary, the blank spaces in the question booklet.
12. **At the end of the examination, the invigilator shall collect the answer sheet first then the question booklet after. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

SECTION A: PURE MATHEMATICS

1. The remainder when the polynomial $f(x) = 2x^3 + ax^2 - 3x + 1$ is divided by $(x + 1)$ is -2. The value of the constant a is
   A 3
   B 4
   C -5
   D -4

2. If the roots of the equation $2x^2 + 5x - 3 = 0$ are α and β, then the value of $\alpha^2 + \beta^2$ is
   A $\frac{29}{4}$
   B $\frac{37}{4}$
   C $\frac{25}{4}$
   D $\frac{31}{4}$

3. If $\frac{x+1}{(x-3)(x-5)} \equiv \frac{M}{x-3} + \frac{N}{x-5}$, then
   A $M = 2$ and $N = 3$
   B $M = -2$ and $N = 3$
   C $M = 3$ and $N = 2$
   D $M = -2$ and $N = -3$

4. The solution set of the inequality $|2x - 1| > 3$ is
   A $\{x: x < -1 \text{ or } x > 2\}$
   B $\{x: x < -2 \text{ or } x > 1\}$
   C $\{x: -1 < x < 2\}$
   D $\{x: -2 < x < 1\}$

5. A partial order relation is that which is
   A reflexive, symmetric and transitive
   B reflexive and anti-symmetric
   C reflexive, anti-symmetric and transitive
   D anti-symmetric and transitive

6. The number of distinct arrangements of the letters of the word ABRACADABRAR is
   A $\frac{12!}{5!\,3!\,2!}$
   B $\frac{12!}{5!\,3!\,3!}$
   C $\frac{12!}{5!\,2!\,2!}$
   D $\frac{12!}{5!\,4!\,3!}$

7. A function $f: \mathbb{R} \to \mathbb{R}$ is given by $f(x) = ax + b$. If $f(3) = 3$ and $f(4) = 5$, then the values of the constants a and b are such that
   A $a = 2, b = 3$
   B $a = -2, b = 3$
   C $a = 2, b = -3$
   D $a = -2, b = -3$

8. The value of $2\log_4 8$ is
   A 2
   B $\frac{3}{2}$
   C 3
   D $\frac{2}{3}$

9. If $u_n = 5 + \frac{3}{2}(n-1)$ is the n^{th} term of an arithmetic progression, the sum of the first twenty terms of the progression is
   A 770
   B 385
   C 358
   D 538

10. The solution set of the inequality $\frac{(1-x)(x-3)}{(x+2)} > 0$ is
    A $\{x: x < -2 \cup 1 < x < 3\}$
    B $\{x: -2 < x < 1 \cup x > 3\}$
    C $\{x: x < 1 \cup 2 < x < 3\}$
    D $\{x: x < -2 \cup x > 3\}$

11. p and q are two statements. The compound statement $p \Longrightarrow q$ is false when
    A p and q have the same truth value
    B p and q are converse of each other
    C p is true and q is false
    D both p and q are false

12. If $a + bi = \frac{1+i}{1-2i}$, the values of the constants a and b are
    A $a = \frac{1}{5}, b = \frac{3}{5}$
    B $a = \frac{1}{5}, b = -\frac{3}{5}$
    C $a = -\frac{1}{5}, b = \frac{3}{5}$
    D $a = -\frac{1}{5}, b = -\frac{3}{5}$

13. If $x^2 - 4x + 9 \equiv (x + A)^2 + B$, then

A $A = 2,\ B = 3$
B $A = 2,\ B = 5$
C $A = -2,\ B = 3$
D $A = -2,\ B = 5$

14. Two variables y and x are connected by the equation $y = a(x + 2)^k$. A linear equivalence of this equation is

A $\log y = \log k(x + 2) + \log a$
B $\log y = a \log(x + 2) + \log k$
C $\log y = k \log 2x + \log a$
D $\log y = k \log(x + 2) + \log a$

15. If matrix $\mathbf{A} = \begin{pmatrix} -1 & k & 2 \\ 2 & -6 & 0 \\ 1 & -2 & -4 \end{pmatrix}$, then the value of k for which $|\mathbf{A}| = 4$ is

A 2
B 3
C -3
D -2

16. If $f: \mathbb{R} \longrightarrow \mathbb{R}$, where $f(x) = \ln(4 + x^2)$, then $f'(2) =$

A $\frac{1}{4}$
B 2
C 4
D $\frac{1}{2}$

17. The period of the trigonometric function $f: \mathbb{R} \longrightarrow \mathbb{R}$, where $f(x) = \sin\frac{2}{3}x$ is

A $\frac{4}{3}\pi$
B $\frac{2}{3}\pi$
C $\frac{3}{4}\pi$
D $\frac{3}{2}\pi$

18. If a polynomial $P(x)$ produces a quotient $(x^2 + 2x + 1)$ and a remainder 5 when divided by $(x + 1)$, then $P(x) =$

A $x^3 + 3x^2 + 3x + 6$
B $x^3 + 3x^2 + 4x + 6$
C $x^3 + 4x^2 + 3x + 6$
D $x^3 + 2x^2 + 3x + 6$

19. The parametric equations of a curve are $1 - x = \tan\theta$ and $= \frac{1}{\cos\theta}$. The cartesian equation of this curve is

A $x^2 - y^2 + 2x + 2 = 0$
B $x^2 - y^2 - x + 2 = 0$
C $x^2 - y^2 - 2x + 2 = 0$
D $x^2 - y^2 + x + 2 = 0$

20. If the cartesian equation of a plane is $x - 2y + z - 5 = 0$, then the vector normal to this plane is

A $\mathbf{i} - 2\mathbf{j} - \mathbf{k}$
B $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$
C $\mathbf{i} - 2\mathbf{j} - \mathbf{k}$
D $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

21. Some values of the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ are given in the table below :

x	2	3	4	5
$f(x)$	5	15	28	k

If by the trapezium rule $\int_2^5 f(x)\,dx \approx 68$, then the value of the constant k is

A 48
B 42
C 47
D 45

22. If x_1 is a first approximate root of the equation $f(x) = 0$. By Newton – Raphson's procedure, a second approximate root of this equation is given by

A $x_1 - \frac{f'(x_1)}{f(x_1)}$
B $x_1 + \frac{f'(x_1)}{f(x_1)}$
C $x_1 - \frac{f(x_1)}{f'(x_1)}$
D $x_1 + \frac{f(x_1)}{f'(x_1)}$

23. A function f is defined by $f(x) = \frac{2x - 3}{x + 1}$, $x \in \mathbb{R}$, $x \neq -1$, the value of $f^{-1}(0)$ is

A $-\frac{3}{2}$
B $\frac{3}{2}$
C $\frac{2}{3}$
D $-\frac{2}{3}$

24. The values of λ and μ for which the lines $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \mu(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ intersect are

A $\lambda = 2$ and $\mu = 1$
B $\lambda = 2$ and $\mu = -1$
C $\lambda = -2$ and $\mu = -1$
D $\lambda = -2$ and $\mu = 1$

25. The number of ways of arranging 5 girls on a bench if the two left-handed girls must be together is

A 24
B 48
C 60
D 96

26. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) =$

A $-\cos x$
B $\sin x$
C $\cos x$
D $-\sin x$

27. The graph of $y = \frac{4}{x^2 - 1}$ has

A two vertical asymptotes $x = \pm 1$ and no horizontal asymptote
B one vertical asymptotes $x = 1$ and one horizontal asymptote $y = 0$.
C two vertical asymptotes $x = \pm 1$ and a horizontal asymptote $y = 4$.
D two vertical asymptotes $x = \pm 1$ and a horizontal asymptote $y = 0$.

28. Given the differential equation $\frac{dy}{dx} - 2xy = 0$. If $y = 1$ when $x = 1$, then

A $y = e^{x^2 - 1}$
B $y = e^{x^2 + 1}$
C $y = e^{x^2}$
D $y = e^{1 - x^2}$

29. The coefficient of x^2 in the binomial expansion of $\left(1 + \frac{1}{2}x\right)^{\frac{1}{2}}$ is

A $\frac{1}{32}$
B $-\frac{1}{16}$
C $\frac{1}{16}$
D $-\frac{1}{32}$

30. The coordinates of the points where the curves $y = 2x - 3$ and $y = x^2 - 2x$ intersect are

A $(1, -1)$ and $(3, 3)$
B $(1, 1)$ and $(3, 2)$
C $(-1, -1)$ and $(3, 3)$
D $(1, -1)$ and $(3, 2)$

31. The value of the constant k for which $\sum_{r=1}^{\infty}\left(\frac{1}{2^{r-1}}\right) = 10$ is

A $\frac{1}{5}$
B 5
C 2
D $\frac{1}{2}$

32. If a linear reduction of the relationship $y = ax^n$ produced the result $\log y = 2\log x + 1.3$, then the approximate values of a and n are

A $a = 2$ and $n = 2$
B $a = 20$ and $n = 2$
C $a = 2$ and $n = 20$
D $a = 20$ and $n = 20$

33. The complex number z, in the form $a + bi$, for which $(1 - 2i)z = 3 - i$ is

A $1 - i$
B $-1 - i$
C $1 + i$
D $-1 + i$

34. The value of a for which the quadratic equation $2x^2 - 6x + 3a = 0$ has equal roots is

A $\frac{2}{3}$
B $-\frac{2}{3}$
C $-\frac{3}{2}$
D $\frac{3}{2}$

35. $\int_1^2 \frac{x}{1+2x^2}\,dx =$

A $4\ln 3$
B $\frac{1}{2}\ln 3$
C $2\ln 3$
D $\frac{1}{4}\ln 3$

SECTION B: MECHANICS

36. A particle has position vector $\mathbf{r}$ at time t seconds given by $\mathbf{r} = (e^{2t}\mathbf{i} + e^{3t}\mathbf{j})$ m. The velocity and acceleration of the particle when $t = 0$ are respectively

A $(2\mathbf{i} + 3\mathbf{j})\ \text{m s}^{-1}$, $(4\mathbf{i} + 9\mathbf{j})\ \text{m s}^{-2}$
B $(2\mathbf{i} + 3\mathbf{j})\ \text{m s}^{-1}$, $(4\mathbf{i} - 9\mathbf{j})\ \text{m s}^{-2}$
C $(3\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$, $(4\mathbf{i} + 9\mathbf{j})\ \text{m s}^{-2}$
D $(2\mathbf{i} - 3\mathbf{j})\ \text{m s}^{-1}$, $(4\mathbf{i} + 9\mathbf{j})\ \text{m s}^{-2}$

37. A particle is projected from a point on a horizontal plane with speed $10\ \text{m s}^{-1}$ at an angle $30°$ to the plane. The horizontal range R covered by the particle is

A $R = 10\sqrt{3}$ m
B $R = \frac{5}{4}\sqrt{3}$ m
C $R = 5\sqrt{3}$ m
D $R = \frac{5}{2}\sqrt{3}$ m

38. When the length of a spring is 60% of its natural length, the thrust in the spring is 10 N. the modulus of elasticity of the spring is

A $\frac{50}{3}$ N
B $\frac{20}{3}$ N
C 25 N
D 4 N

39. A centripetal force of 72 N causes a particle of mass 4 kg to move in a horizontal circle of radius 2 m. The angular speed of the particle is

A $9\ \text{rad s}^{-1}$
B $3\ \text{rad s}^{-1}$
C $9\ \text{m s}^{-1}$
D $3\ \text{m s}^{-1}$

40. Two particles of masses 2 kg and 3 kg are placed at the points (3, 1) and (8, 6) respectively. The position of their center of mass is at the point

A (6, 4)
B (5·5, 3·5)
C (5, 3)
D (4, 6)

41. Fig. 1 shows a uniform sheet of metal in the form of letter L with dimensions in metres.

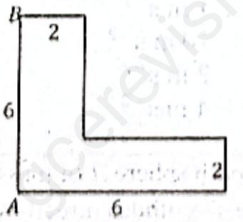

Fig. 1

The distance of its center of mass from edge AB is

A 2.2 m
B 1.8 m
C 3 m
D 2.5 m

42. A uniform ladder AB, of length $2a$, rests limiting equilibrium with its top end against a rough vertical wall and its lower end on rough horizontal floor. The forces acting on the ladder are as shown in fig. 2 below,

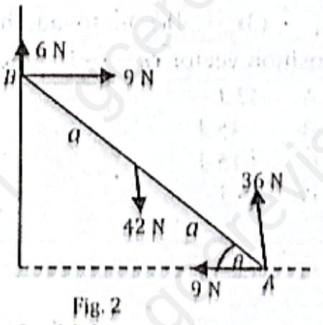

Fig. 2

The angle θ which AB makes with floor is given by

A $\tan\theta = \frac{3}{5}$
B $\tan\theta = \frac{1}{5}$
C $\sin\theta = \frac{3}{5}$
D $\sin\theta = \frac{1}{5}$

43. A girl on a ship, whose velocity is $(4\mathbf{i} - 7\mathbf{j})$ m s^{-1} is watching a boat whose velocity is $(-8\mathbf{i} - 2\mathbf{j})$ m s^{-1}. The speed of the boat as it appears to the girl on the ship is

A $(-12\mathbf{i} + 5\mathbf{j})$ m s^{-1}
B $(12\mathbf{i} - 5\mathbf{j})$ m s^{-1}
C 13 m s^{-1}
D $(\sqrt{68} - \sqrt{65})$ m s^{-1}

44. A particle accelerating from rest has its speed v ms^{-1} at time t seconds given by

$$v = 6t - \frac{1}{2}t^2.$$

The acceleration of the particle when $v = 10$ is

A 1 m s^{-2}
B 10 m s^{-2}
C 2 m s^{-2}
D 4 m s^{-2}

45. A smooth sphere A of mass 4 kg travelling at 5 m s^{-1} collides directly with a smooth sphere B of mass 3 kg travelling at 4 m s^{-1} in the opposite direction. Given that the speed of A after impact is 0.5 m s^{-1}, the kinetic energy of B after impact is

A 24 J
B 3 J
C 6 J
D 6.5 J

46. The work done by a force $\mathbf{F} = (\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ N which moves its point of application from a point A with position vector $\mathbf{r}_A = (3\mathbf{i} - 4\mathbf{k})$ m to another point B with position vector $\mathbf{r}_B = (-\mathbf{i} + 6\mathbf{k})$ m is

A 47 J
B −35 J
C −15 J
D 35 J

47. The engine of a car of mass 1,200 kg works at a constant rate of 54 kW up a road inclined at an angle $\sin^{-1}\left(\frac{1}{5}\right)$ to the horizontal. Given that the non-gravitational resistance to the motion of the car is 300 N and taking g as 10 m s^{-2}, the maximum speed of the car is

A 20 m s^{-1}
B 25 m s^{-1}
C 100 m s^{-1}
D 900 m s^{-1}

48. A block of mass 4 kg slides down a rough plane inclined at 30 to the horizontal at constant speed. The block covers a distance 0.8 m. Taking g as 10 m s^{-2}, the work done against friction is

A 16 kJ
B 16 J
C 32 J
D 32 kJ

49. From a speed of 20 m s^{-1}, a car decelerates uniformly to rest, covering a distance of 80 m. The deceleration of the car is

A $-\frac{5}{2}$ m s^{-2}
B $\frac{5}{2}$ m s^{-2}
C 8 m s^{-2}
D $-\frac{2}{5}$ m s^{-2}

50. Two dice are thrown together. The probability of having at least a SIX is

A $\frac{11}{36}$
B $\frac{1}{18}$
C $\frac{1}{36}$
D $\frac{1}{3}$

GO BACK AND CHECK YOUR WORK