



1.  $\int \sinh^2 x dx =$
- A.  $\frac{1}{4}(\sinh 2x + 2x) + C$
  - B.  $\frac{1}{4}(\sinh 2x - 2x) + C$
  - C.  $\frac{1}{4}(\cosh 2x + 2x) + C$
  - D.  $\frac{1}{4}(\sinh 2x + x) + C$

2. The polar equation  $r = -8\cos x$  represents
- A. a circle
  - B. a hyperbola
  - C. a parabola
  - D. an ellipse

3. The values of  $u$  for which
- $$\begin{vmatrix} 6-u & 0 & 1 \\ -1 & -6-u & -2 \\ 5 & 0 & u \end{vmatrix} = 0$$
- are
- A.  $-1, -5, -6$
  - B.  $1, -5, 6$
  - C.  $-1, 5, -6$
  - D.  $1, 5, -6$

4. If a particle moves along a straight line  $OX$  such that its displacement  $x$  meters from  $O$  at time  $t$  seconds is given by the differential equation  $\frac{d^2y}{dx^2} + x = 0$ , then its period of motion is
- A.  $\frac{\pi}{3}$
  - B.  $\frac{2\pi}{\sqrt{3}}$
  - C.  $\frac{\pi}{2}$
  - D.  $2\pi$

5. The distance between the planes  $x + y - z = 3$  and  $x + y - z = -6$  is
- A.  $3\sqrt{3}$
  - B. 9
  - C. 3
  - D.  $\sqrt{3}$

6. If  $f(x) = \int_a^x \frac{2t}{\sqrt{t^2+1}} dt$ , then  $a \geq 0$ ,  $f$
- A. is concave downwards
  - B. is monotone increasing
  - C. has a turning point
  - D. is monotone decreasing

7. A group  $H$  has a subgroup  $\{x, y\}$ ,  $\{x, y, w, u, v\}$  and  $\{x, w\}$ . The identity element of  $H$  is:
- A.  $w$
  - B.  $y$
  - C.  $x$
  - D.  $u$

8. Which of the following series is convergent?
- A.  $\sum_{n=1}^{\infty} \frac{1}{n}$
  - B.  $\sum_{n=0}^{\infty} \frac{3^n}{3^n}$
  - C.  $\sum_{n=0}^{\infty} (1-n)^n$
  - D.  $\sum_{n=0}^{\infty} n(1+n)$

9. A particle moves round a polar curve  $r = a(1 + \cos\theta)$  with constant angular velocity  $\omega$ . The transverse component of its velocity is:
- A.  $a\omega(1 - \sin\theta)$
  - B.  $-a\omega \sin\theta$
  - C.  $\omega$
  - D.  $a\omega(1 + \cos\theta)$

10. If  $\omega$  is the complex cube root of unity, the value of  $\omega^4 + \omega^8$  is.
- A.  $-1$
  - B. 1
  - C.  $-2$
  - D. 2

11.  $\int \frac{1}{9x^2+4} dx =$
- A.  $\frac{1}{3} \tan^{-1} \left( \frac{3x}{2} \right) + C$
  - B.  $\frac{1}{6} \tan^{-1} \left( \frac{3x}{4} \right) + C$
  - C.  $\frac{1}{3} \tan^{-1} \left( \frac{3x}{5} \right) + C$
  - D.  $\frac{1}{6} \tan^{-1} \left( \frac{3x}{2} \right) + C$

12. Which of the following is not correct?
- A. Differentiability implies continuity
  - B. Continuity implies differentiability
  - C. Conflexion implies continuity
  - D. Conflexion implies differentiability

13. Given the sequence  $U_n = \ln(n^2 + 1) - 2\ln n$ , for all natural numbers  $n, n \neq 0$ , then the lower and upper bounds are respectively
- A. 0 and 1
  - B. 1 and 2
  - C. 0 and  $\ln 2$
  - D. 1 and  $\ln 2$

14. The type of curve represented by the equation  $x^2 + xy + y^2 - 1 = 0$  is:
- A. an ellipse
  - B. a hyperbola
  - C. a parabola
  - D. a circle

15. A sequence is defined recursively as  $a_1 = a_2 = 1, a_n = a_{n-1} + a_{n-2}, n \geq 3$ . The seventh term is:
- A. 8
  - B. 21
  - C. 5
  - D. 13

16. Given the equation of a hyperbola as  $b^2x^2 - a^2y^2 = a^2b^2$ , then the equations of the asymptotes are:
- A.  $y = \pm \frac{a}{b}x$
  - B.  $y = \pm \frac{b}{a}x$
  - C.  $x = \pm \frac{a}{b}y$
  - D.  $x = \pm \frac{b}{a}y$

17. The parametric equations of the rectangular hyperbola  $x(y+1) = c^2$  are:
- A.  $x = ct, y = \frac{c}{t} - 1$
  - B.  $x = ct, y = \frac{t}{c} + 1$
  - C.  $x = 1 + ct, y = \frac{c}{t}$
  - D.  $x = ct, y = \frac{c}{t}$

18. The mean value of  $\frac{1}{1+4x^2}$  for  $0 \leq x \leq \frac{1}{2}$  is:
- A.  $\frac{\pi}{2}$
  - B.  $\frac{\pi}{4}$
  - C.  $\frac{\pi}{8}$
  - D.  $\frac{\pi}{16}$

19. Given that  $P$  and  $Q$  are  $3 \times 3$  invertible matrices, then  $(P^{-1}Q)^{-1} =$
- $QP^{-1}$
  - $Q^{-1}P^{-1}$
  - $Q^{-1}P$
  - $PQ^{-1}$

20. Given that  $\lim_{x \rightarrow 0} \frac{\sqrt{1+f(x)}}{x} = l$ , then

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+f(3x)}}{x} =$$

- $3l$
- $\sqrt{3}l$
- $\frac{1}{2}l$
- $\frac{1}{3}l$

21. If  $\vec{a} = i + j$ ,  $\vec{b} = 2i - j$ , then  $\vec{a} \times \vec{b} =$

- $0\vec{k}$
- $0$
- $\vec{i}$
- $-3\vec{k}$

22. The cofactor of the element 6 in  $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$  is:

- $\begin{vmatrix} 7 & 8 \\ -1 & -2 \end{vmatrix}$
- $\begin{vmatrix} -1 & -2 \\ 7 & 8 \end{vmatrix}$
- $\begin{vmatrix} -1 & -2 \\ -7 & -8 \end{vmatrix}$
- $\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$

23. The equations  $\begin{cases} x - 2y + 4z = 0 \\ 2x - 4y - 3z = 0 \end{cases}$

- are inconsistent
- represent a pair of intersecting lines
- have a unique solution
- have solutions depending on one parameter

24. If  $f(x) = \tanh x$ , then  $f(x)$  is

- periodic
- discontinuous
- continuous
- even

25. The equation  $5x^3 - 9x^2 + 12x + 4 = 0$

- has roots  $\alpha, \beta$  and  $\delta$  where  $\alpha\beta\delta = \frac{2}{5}$
- has three real roots
- has a repeated root
- has a root between 0 and 1

26. If the period of a function  $f(x)$  is  $T$ , then the period of  $f(2x + k)$ , where  $k$  is real is:

- $2T$
- $\frac{1}{2}T$
- $\frac{4}{T}$
- $\frac{1}{T}$

27. Given that  $f(x) = x^2 + 2x$  and  $g(x) = 3x^3 + 6x^2$ ,  $x > 0$ , the gcd for  $f$  and  $g$  is:

- $x$
- $x + 2$
- $x^2 + 2x$
- $3x + 6$

28. The moment of inertia of an object of mass  $2m$  is  $\frac{8ma^2}{3}$ . Its radius of gyration is:

- $\frac{4a^2}{3}$
- $\frac{1}{2a}\sqrt{3}$
- $\frac{2a}{2\sqrt{3}}a$
- $\frac{a^3}{3}\sqrt{3}$

29. The integrating factor of the differential equation

$$\frac{dy}{dx} + \frac{2y}{x} = 1 \text{ is } I =$$

- $x^2$
- $\frac{1}{x}$
- $-\frac{1}{x}$
- $-x^2$

30. The number  $x$ , of bacteria in a culture decreases at a rate equal to twice the number of bacteria present at time  $t$ . The differential equation describing this situation where  $k$  is a positive constant is:

- $\frac{dx}{dt} - 2kx = 0$
- $\frac{dx}{dt} + 2kx = 0$
- $\frac{dx}{dt} = kx$
- $\frac{dx}{dt} = -2k$

31. The equation  $\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + n^2x = 0$  represents a damped harmonic motion if:

- $k^2 = n$
- $k^2 < n^2$
- $2k = n^2$
- $k^2 > n^2$

32.  $\int \frac{1}{\sqrt{x^2+3x-1}} dx =$

- $\cosh^{-1} \frac{3}{\sqrt{13}} \left(x + \frac{3}{2}\right) + k$
- $\sqrt{\frac{4}{13}} \sinh^{-1} \left(x + \frac{3}{2}\right) + k$
- $\sinh^{-1} \left(x + \frac{3}{2}\right) + k$
- $\sqrt{\frac{14}{13}} \cosh^{-1} \left(x + \frac{3}{2}\right) + k$

33. The work done by a force  $\mathbf{F}$  in moving a particle of mass  $m$  from a point A with position vector  $\mathbf{a}$  to a point B with position vector  $\mathbf{b}$  is

- $\mathbf{F} \times (\mathbf{b} - \mathbf{a})$
- $m\mathbf{F} \cdot (\mathbf{b} - \mathbf{a})$
- $\mathbf{F} \cdot (\mathbf{b} - \mathbf{a})$
- $m\mathbf{F} \times (\mathbf{b} - \mathbf{a})$

34. The equation of the asymptote of the curve  $x^3 + y^3 - 6x^2 = 10$  is:

- $y = x + 2$
- $y = -x + 2$
- $y = x$
- $y = -x$

35.  $\sum_{k=1}^{2n} (-1)^k =$

- $0$
- $\infty$
- $1$
- $-1$

36. The range of the function  $y = \frac{x}{x^2+1}$  for real values of  $x$  is:

- A.  $-\frac{1}{2} \leq y < \frac{1}{2}$   
 B.  $-\frac{1}{2} \leq y \leq \frac{1}{2}$   
 C.  $y < -\frac{1}{2}, y > \frac{1}{2}$   
 D.  $y \leq -\frac{1}{2}, y \geq \frac{1}{2}$

37. The area under the curve with parametric equations  $x = 1 + t^2, y = t(2 - t)$  in the interval  $0 \leq t \leq 1$  is:

- A.  $\frac{5}{6}$   
 B.  $\frac{6}{5}$   
 C.  $\frac{2}{3}$   
 D.  $\frac{5}{3}$

38. A smooth sphere travelling with a speed of  $2m/s$  on a smooth horizontal floor hits a vertical wall at an angle of  $45^\circ$ . Given that the coefficient of restitution is  $\frac{1}{4}$ , then its speed after impact is:

- A.  $\frac{1}{4}\sqrt{3} m/s$   
 B.  $\frac{3}{4}\sqrt{2} m/s$   
 C.  $\sqrt{17} m/s$   
 D.  $\frac{1}{4}\sqrt{34} m/s$

39.  $\frac{5x^3}{(x^2+x+1)(x-2)}$  in partial fractions, where  $A, B, C$  and  $D$  are real constants is:

- A.  $\frac{Ax+B}{x^2+x+1} + \frac{C}{x-2}$   
 B.  $\frac{Ax}{x^2+x+1} + \frac{B}{x-2} + D$   
 C.  $\frac{Ax+B}{x^2+x+1} - \frac{C}{x-2}$   
 D.  $\frac{Ax+B}{x^2+x+1} + \frac{C}{x-1} + D$

40. If  $P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$  and  $Q = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$  are permutations of the elements  $(1, 2, 3, 4)$ , then  $QP =$

- A.  $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$   
 B.  $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$   
 C.  $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$   
 D.  $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

41. If  $\omega = iz$  is a transformation from the  $z$  to the  $\omega$  complex planes, then the invariant point is:

- A. 1  
 B.  $i$   
 C. 0  
 D. -1

42. Given that  $f$  is the probability density function, where

$$f(x) = \begin{cases} \frac{3x^2}{k}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

the value of  $k$  is:

- A. 1  
 B. 2

C. 27

D. 8

43. The series  $\sum_{n=1}^{\infty} \left(\frac{5}{2n}\right)^r$  converges when

- A.  $r < 1$   
 B.  $r = 1$   
 C.  $r = 0$   
 D.  $r > 1$

44. The center of symmetry of the curve  $y = \frac{1}{x-2}$  is at the point;

- A. (2,0)  
 B. (0,2)  
 C. (0,0)  
 D. (-2,0)

45. The particular solution of the differential equation  $\frac{d^2y}{dx^2} + 9y = \sin 3x$ , where  $a$  and  $b$  are constants is of the form

- A.  $y = ax \sin 3x$   
 B.  $y = a \sin x + b \cos x$   
 C.  $y = ax \cos 3x$   
 D.  $y = a \sin 3x + b \cos 3x$

46. The vertices  $A, B, C$  of a triangle have the position vectors  $\mathbf{a}, \mathbf{b}$  and  $\mathbf{c}$  respectively. The area of this triangle is:

- A.  $|\mathbf{a} \times \mathbf{b} \cdot \mathbf{c}|$   
 B.  $\frac{1}{2}|(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{b})|$   
 C.  $|(\mathbf{b} - \mathbf{a}) \times (\mathbf{b} - \mathbf{c})|$   
 D.  $\frac{1}{2}|\mathbf{b} \times \mathbf{c}|$

47. If  $T = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ , the image of the line  $y = -2x$  under  $T$  is:

- A.  $y = 0$   
 B.  $x = 0$   
 C.  $y = \frac{x}{3}$   
 D.  $y = 3x$

48. If the probability density function  $f$  of  $X$  is

$$f(x) = \begin{cases} \frac{3x^2}{a}, & 0 \leq x \leq a \\ 0, & \text{otherwise} \end{cases}$$

- A.  $\frac{3x^3}{a^3}$   
 B.  $\frac{3a^2}{4}$   
 C.  $\frac{3a}{4}$   
 D.  $\frac{3}{4}$

49. The integrating factor for the differential equation  $x \frac{dy}{dx} - y = x$  is:

- A.  $-x$   
 B.  $\frac{1}{x}$   
 C.  $x$   
 D.  $-\frac{1}{x}$

50. If a force  $F$  acts through a point with position vector  $\mathbf{a}$ , then its moment about a point with position vector  $\mathbf{b}$  is:

- A. 0  
 B.  $\mathbf{a} \times \mathbf{F}$   
 C.  $(\mathbf{a} - \mathbf{b}) \times \mathbf{F}$   
 D.  $(\mathbf{b} - \mathbf{a}) \times \mathbf{F}$

END