REPUBLIQUE DU CAMEROUN
Paix-Travail-Patrie

MINISTERE DES ENSEIGNEMENTS SECONDAIRES

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TEACHERS' RESOURCE UNIT TRU-RIP-STA MOCK EXAMS NORTH WEST REGION

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REPUBLIC OF CAMEROON
Peace-Work-Fatherland

MINISTRY OF SECONDARY EDUCATION

TEACHERS' RESOURCE UNIT
REGIONAL BRANCH FOR THE NORTH WEST

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MARCH 2023

The Teachers' Resource Unit and the Regional Inspectorate of Pedagogy in collaboration with MTA	SUBJECT CODE NUMBER **0775**	PAPER NUMBER **2**
GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION	SUBJECT TITLE **FURTHER MATHEMATICS**	
ADVANCED LEVEL		

Time Allowed: THREE hours

INSTRUCTIONS TO CANDIDATES

NSINANG

Mobile phones are **NOT ALLOWED** in the examination room.

Full marks may be obtained for answers to **ALL** questions.
In calculations, you are advised to show all the steps in your working, giving your answer at each stage.
Non programmable electronic calculators, Mathematical formulae and tables are allowed.

You are reminded of the necessity for good English and orderly presentation in your answers.

TURN OVER

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1. Show that $\int_0^{\sqrt{2}} \sqrt{1+4x^2}\,dx = \frac{3}{2}\sqrt{2} + \frac{1}{4}\ln\left(3+2\sqrt{2}\right)$. 4 marks

Hence or otherwise, find the length of arc L of the parabola $y^2 = x$, for which $0 \le y \le \sqrt{2}$ 3 marks
Further show that the area of the surface generated when the arc L is rotated completely about the $x-$ axis is $\frac{39}{4}\pi$. 4 marks

2. Given the differential equation

$$\frac{d^2y}{dx^2} - 4y = 2e^{-2x} + 1.$$

Find the complementary function of this differential equation. 2 marks
Hence find the solution of the differential equation for which when $x = 0$, $y = 0 = \frac{dy}{dx}$. 6 marks

3. a)Solve completely the equation justifying your results
$4\tanh x - \sinh 2x = 0$. 3 marks
Given that $y = \sinh(\frac{1}{2}x)$,
prove by Mathematical induction that

$$\frac{d^{2n}y}{dx^{2n}} = 4^{-n}\sinh(\tfrac{1}{2}x),\ n \in \mathbb{N}.$$ 4 marks

4. Find the $\gcd(40321, 35287)$. 2 marks
Hence find the values of a and b for which
$40321a + 35287b = 1$. 3 marks
c) Solve the congruence
$7x \equiv 2 (\bmod 15)$ 1 mark
Hence, using the Chinese remainder theorem or otherwise solve the system
$7x \equiv 2 (\bmod 15)$
$3x \equiv 1 (\bmod 4)$ 3 marks

5. The Cartesian equation of a Hyperbola H is

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

Find
i) the foci and the directrice of H. 3 marks
ii) the equation of the tangent at the point $P(x_1, y_1)$ 2 marks
iii) the equation of the normal at P. 2 marks
The normal cuts the coordinate axes at A and B, while the tangent cuts at C and D.
iv) Find the ratio of the area $\Delta OAB : \Delta OCD$. 4 marks

6a. Show that the parabola $x^2 = 2y$ is represented by the polar curve,

$$r = \frac{2\sin\theta}{1-\sin^2\theta}.$$ 3 marks

Find the equation of the tangent to this curve that is parallel to the initial line. 2 marks
b. A complex transformation from the $z-$plane to the $w-$ plane is defined by

$$w = \frac{2+i}{i+z}$$

i) Show that $z = \frac{2+i-iw}{w}$. 1 mark
ii) Find the invariant points of the transformation in the form $a+bi,\ a, b \in \mathbb{Q}$. 3 marks
iii) Show that the image of the circle $|z| = 1$ is a line in the $w-$ plane, and describe the line fully. 3 marks

7. A transformation T is defined by the matrix M where

$$M = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}.$$

Find

i) $|M|$. Hence state the invariant point and justify your answer. 2 marks

ii) M^{-1} 2 marks

iii) the plane whose image is

$$x + 3y - z = 0.$$ 3 marks

8. A plane Π_1 contains three non-collinear points A, B and C with position vectors

$$\mathbf{a} = \mathbf{i} - 3\mathbf{j}, \quad \mathbf{b} = \mathbf{i} - \mathbf{k}, \quad \mathbf{c} = 3\mathbf{j} + \mathbf{k} \text{ respectively.}$$

i). Find the area of triangle ABC. 2 marks

ii). Find the equation of Π_1 in the form $\mathbf{r}\cdot\mathbf{n} = d$. 2 marks

A second plane Π_2 has equation $\mathbf{r}\cdot(\mathbf{i} - 3\mathbf{k}) = d$.

iii). A plane Π is perpendicular to both Π_1 and Π_2. Find the equation of the plane Π. 4 marks

9. A function f is defined by

$$f(x) = \frac{\ln(x-1)}{x-2}.$$

Find

i) the domain of f. 2 marks

ii) the following $\lim_{x \to b} f(x)$, where b is every boundary of the dom(f) 4 marks

iii) Find $f''(x)$ 2 marks

iv) Given that $f''(x) > 0$ in its domain, state the relative position of any tangent drawn to the curve $y = f(x)$. 1 mark

v) Draw the table of signs of f. 3 marks

vi) Sketch the curve $y = f(x)$. 2 marks

10. Two recursive sequences are defined as:

$$u_1 = 1, \qquad u_{n+1} = u_n - 3,$$
$$\text{and } v_1 = 1, \qquad v_{n+1} = 4v_n$$

Find

i). u_n and v_n in terms of n. 3 marks

Another sequence is defined by

$$w_n = \frac{1 - u_n}{v_n}.$$

Show that

ii). $w_n \geq 0, \ \forall n \in \mathbb{N}$. 3 marks

iii). w_n is bounded 3 marks

Find

iv). real numbers a and b such that

$$a \leq w_n \leq b.$$ 3 marks

END

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