0770/2/2023
P.M.S. A/L

SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION

THE TEACHERS' RESOURCE UNIT (TRU)

IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR SCIENCE

AND

THE SOUTH-WEST ASSOCIATION OF MATHEMATICS TEACHERS

(SWAMT)

TUESDAY: 28/03/2023-MORNING ADVANCED LEVEL

Subject Title	Pure Mathematics With Statistics
Paper Number	Paper 2
Subject Code Number	0770

THREE HOURS

INSTRUCTIONS TO CANDIDATES:

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the CGCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage. .

1(i) The coefficient of x^3 in the expansion of $P(x)$, where $P(x) = (5 + 2x^2)(x^3 + kx + m)$ is 29.

Also, the remainder when $x^3 + kx + m$ is divided by $x - 3$ is 59.

Find the values of the constants k and m. (5 marks)

(ii) The function f is defined by $f(x) = x^2 + 2kx + 3k + 4 = 0$, where k is a constant .

(a) Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants. (2 marks)

Hence or otherwise,

(b) find the range of values of k for which the equation $f(x) = 0$ has no real solutions. (4 marks)

2(i) (a) Prove that $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ (4 marks)

Hence,

(b) Find the value of θ given that $\tan^{-1} 4 - \tan^{-1}\left(\frac{3}{5}\right) = \theta - \frac{\pi}{4}$ (2 marks)

(ii) Find the general solution of the equation $\cos 2\theta = \sin\theta$ (3 marks)

3(i)

t	0	5	10	15	20	25	30
v	0	1.22	2.28		6.11		

The table above shows the speed of a train v at intervals of 5 seconds. Given that the speed is related to the time by the equation $v = \sqrt{1.2^t - 1}$, $0 \le t \le 30$,

(a) Copy and complete the table giving your values to 2 decimal places. (3 marks)

The distance S meters travelled by the train in 30 seconds is given by $S = \int_0^{30} (\sqrt{1.2^t - 1})dt$.

(b) Using the trapezium rule with all the values from your table, estimate the value of S. (3 marks)

(ii) A curve is defined by the equation $\sin 2x \cos x + \cos y = \sqrt{2}$.

(a) Verify that the point $P\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ lies on the curve. (1 mark)

(b) Find $\frac{dy}{dx}$ in terms of x and y. Hence, find the gradient of the curve at the point P. (4 Marks)

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4. With respect to a fixed origin, O, the lines l_1 and l_2 are given by the equations

$$l_1: r = 11i + 2j + 17k + \lambda(-2i + j - 4k)$$
$$l_2: r = -5i + 11j + pk + \mu(qi + 2j + 2k),$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) Show that $q = -3$. (2 marks)

Given further that l_1 and l_2 intersect, find

(b) the value of p and state the position vector of the point of intersection. (6 marks)

(c) Given that the point $(m, 6, n)$ is on l_1, find the values of the constants m and n. (3 marks)

5(i) Given that $w = \frac{z-1}{\bar{z}+1}$, where $z = x + yi$ and $\bar{z}$ is the conjugate of z, show that if w is purely imaginary, then $x^2 - 1 - y^2$ (4 marks)

(ii) The complex numbers z_1 and z_2 are given as $z_1 = 1 - i$ and $z_2 = 1 + \sqrt{3}i$. Evaluate

(a) $\arg(z_2{}^4)$ (3 marks)

(b) $|z_1 z_2|$ (2 marks)

6(i) The function f is defined on $\mathbb{R} - \{1\}$ as $f(x) = \frac{5-x}{x-1}$

(a) Determine the points where the curve of f crosses the coordinate axes (1 mark)

(b) Find the asymptotes of the curve of $f(x)$ (2 marks)

(c) Show that $f(x)$ is always decreasing (2 marks)

Hence,

(d) Sketch the graph of f, showing clearly its intercepts and the behavior of the curve as it approaches the asymptotes. (2 marks)

(ii) Solve the following equations simultaneously for x and y

$$\log x - \log y = 1 \text{ and } x + y^2 = 200$$ (5 marks)

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7 (i) A relation $\mathcal{R}$ is defined on $\mathbb{N}$, the set of natural numbers by

$a\mathcal{R}b$ if and only if "$a + b$ is even".

Show that $\mathcal{R}$ is an equivalence relation. (4 marks)

(ii) The sets A and B are defined by $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f: A \longrightarrow B$ is define by $f(x) = \frac{x-2}{x-3}, \forall x \in A$. Prove that f is bijective. (5 marks)

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8. (a) By using the substitution $u = 1 + \sqrt{x}$, show that $\int_1^4 \frac{2}{\sqrt{x}(1+\sqrt{x})} dx = 4 \ln\left(\frac{3}{2}\right)$ (6 marks)

(b) Obtain in the form $y = f(x)$ a particular solution of the differential equation

$(x^2 + 4)\frac{dy}{dx} = 2xy$ given that $y = 4$ when $x = 0$ (4 marks)

9(i) The 10^{th} term of an Arithmetic progression is twice the 4^{th} term. The 20^{th} term of the progression is 44. Find the common difference of the progression. (4 marks)

(ii) If p is the statement: "*Salah is a fantastic footballer*" and q the statement:

"*A fantastic footballer deserves the Best Player Award*", write down the statement represented by

(a) $p \Longrightarrow q$ (1 mark)
(b) $p \vee q$ (1 mark)
(c) $\sim(p \wedge q)$ (1 mark)
(d) The converse of $q \Longrightarrow p$ (1 mark)

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10 (i) Given that A and B are invertible matrices, show that $(AB)^{-1} = B^{-1}A^{-1}$ (3 marks)

(ii) The matrices M and N are given as $M = \begin{pmatrix} -6 & 2 & 1 \\ 5 & -2 & -1 \\ 3 & -1 & -1 \end{pmatrix}$ and $N = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & -1 \\ 1 & 0 & 2 \end{pmatrix}$.

(a) Find the matrix products MN and NM and state the relationship between M and N (6 marks)

Hence,

(b) solve the system of equations

$$-6x + 2y + z = 5$$
$$5x - 2y - z = -3$$
$$3x - y - z = 4$$

(3 marks)

END.