ADDITIONAL MATHEMATICS 2

0575/2

WEST MATHEMATICS TEACHERS' PEDAGOGIC GROUP

GENERAL CERTIFICATE OF EDUCATION MOCK EXAMINATION

30th MARCH, 2023 **ORDINARY LEVEL**

Subject Title	**Additional Mathematics**
Paper No.	**Paper 2**
Subject Code No.	**0575**

Two and a half hours

Answer ALL QUESTIONS IN SECTION A and ANY TWO QUESTIONS FROM EITHER SECTION B or SECTION C. IN SECTION B AND C, ALL QUESTIONS CARRY EQUAL MARKS.

Candidates are expected to answer a combination of **Section A** and **Section B OR Section A** and **Section C** but **NOT** a combination of all three.

All necessary working must be shown. No marks will be awarded for answers without brief statements showing how the answers have been obtained.

Electronic calculators are allowed

Where necessary take g as $10ms^{-2}$.

SECTION A: PURE MATHEMATICS
THIS SECTION IS COMPULSORY TO ALL CANDIDATES
(ANSWER ALL QUESTIONS)

1. (i) Given that $(x+2)$ is a factor of $f(x)$ where $f(x) = x^3 - x^2 + kx + 4$.

   (a) Find the value of k. **(2 marks)**

   With this value of k,

   (b) factorize $f(x)$ completely. **(2 marks)**

   (ii) Find the value of p for which the equation $px^2 + x + 1 = 0$ has equal roots. **(2 marks)**

   (iii) Given that α and β are the roots of the equation $x^2 + 3x + 1 = 0$, show that $\alpha - \beta = \sqrt{5}$.

   (2 marks)

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2. (i) Find the number of permutations of the letters of the word "ABOUBAKAR".

   (3 marks)

   (ii) Find the numerical value of the term independent of x in the binomial expansion of $\left(2x - \frac{1}{x}\right)^{10}$.

   (4 marks)

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3. (i) The first term of an AP is a and the common difference is d where $d > 0$. Given that the sum of the 1^{st} and 2^{nd} terms is x and the sum of the 9^{th} and 10^{th} terms is y.

   (a) Find $x + y$ in terms of a and d.

   (2 marks)

   (b) Show that the sum of the first 10 terms of the progression is $\frac{5}{2}(x+y)$. **(3 marks)**

   (ii) Express in the form $a + b\sqrt{15}$, where a and b are real numbers, the fraction $\frac{2\sqrt{5} + 3\sqrt{3}}{3\sqrt{5} - \sqrt{3}}$.

   (3 marks)

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4. (i) The set $S = \{a, b, c, d\}$ under the operation $*$ has the following properties:
   - d is the identity element,
   - the operation $*$ is commutative over S,
   - the set S is closed under $*$,
   - $a * c = b, \quad b * a = c, \quad b * c = d$

   (a) Draw an operation table for the set S under the operation $*$. **(3 marks)**

   (b) State the inverse of each element. **(2 marks)**

   (ii) The linear transformation, T, is defined as $T : (x, y) \mapsto (2x - y, \ -x + 2y)$. Find

   (c) the image of the point $(5, -1)$ under the transformation T. **(2 marks)**

   (d) the invariant line under the transformation T. **(2 marks)**

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5. A form five student is to register x practical and y non-practical subjects for the GCE O-level examination. The student must register at most 11 subjects and the non-practical subjects must be more than the practical subjects.
Given that the student is to register;

at least 3 non-practical subjects,

at least 1 practical subject.

(a) Write down four inequalities in terms of x and y that satisfy these conditions.

(3 marks)

(b) On a graph paper, taking 1 cm to represent 1 unit on both axes, shade, so as to leave unshaded, the region represented by these 4 inequalities. **(3 marks)**

Given that the registration fee is 11 000 FCFA and that a practical and non-practical subject cost 5 000 FCFA and 1 000 FCFA respectively.

(c) Find the minimum amount that the student can possibly spend for the GCE registration.

(2 marks)

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6. (i) Show that $(\cot^2\theta)(1-\cos^2\theta) \equiv \cos^2\theta$. **(2 marks)**

(ii) The function f is defined as follows:

$$f(x) = \sin x - \cos x \quad \text{for} \quad 0^\circ \leq x \leq 180^\circ$$

(a) Copy and complete the table below. **(2 marks)**

x	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
y		-0.7			0.4			1.2			1.4		

(b) Taking 1 cm to represent 15° on the $x-$axis and 4 cm to represent 1 unit of the $y-$axis, draw the graph of $y = f(x)$. **(3 marks)**

(c) By drawing a suitable straight line on the graph, find the values of x for which $\sin x - \cos x = \frac{1}{2}$. **(2 marks)**

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7. Given the lines l_1 and l_2 where

$$l_1 : \mathbf{r}_1 = \mathbf{i} - \mathbf{j} + s(2\mathbf{i}+\mathbf{j})$$
$$l_2 : \mathbf{r}_2 = 3\mathbf{i} - 2\mathbf{j} + t(2\mathbf{i} - 2\mathbf{j})$$

Find,

(a) the position vector of the point of intersection of l_1 and l_2 **(5 marks)**

(ib) the cosine of the angle between l_1 and l_2. **(3 marks)**

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8. (i) Differentiate with respect to x the function $f(x) = \dfrac{(\pi + x^2)\cos x}{\sin \pi}$. **(2 marks)**

(ii) Evaluate; $\int (1 - \sin x)\, dx$. **(2 marks)**

(iii) The gradient of the real valued function $y = ax^2 + bx - 6$ when $x = 2$ is 8. Given that the function passes through the point $(2, 7)$, find the values of a and b. **(4 marks)**

SECTION B: MECHANICS

IF THIS SECTION IS CHOSEN, THEN SECTION C MAY NOT BE CHOSEN

(ANSWER ANY TWO QUESTIONS)

(Where necessary, take the acceleration due to gravity, $g = 10\ \text{ms}^{-2}$)

9. (i) Two particles A and B move in the $x - y$ plane such that at time t seconds, A and B have position vectors $[(3t^2 - 1)\mathbf{i} + (t^2 + t - 1)\mathbf{j}]$ m and $[t^2\mathbf{i} + (4t - 2)\mathbf{j}]$ m respectively.

(a) Calculate the distance between A and B when $t = 2$. **(3 marks)**

(b) Find the magnitude and direction when $t = 2$ of the velocity of A relative to B. **(6 marks)**

(c) Find the acceleration of A at $t = 2$. **(2 marks)**

(ii) Figure 1 below shoes a wedge with a smooth surface and two masses connected by a string passing over a smooth fixed pulley. Given that the system is released from rest with the string taut, find;

(d) the common acceleration of the masses, **(4 marks)**

(e) the tension in the string. **(2 marks)**

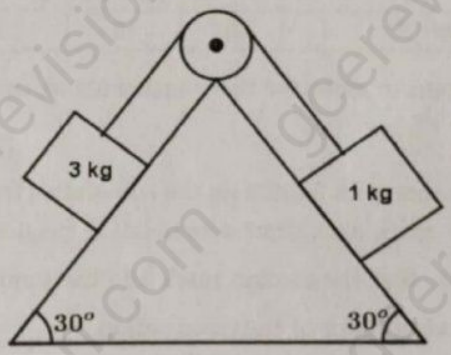

Figure 1

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10. (i) Gas is escaping from a spherical balloon at the rate of $2\ \text{cm}^3\text{s}^{-1}$. Find the rate of decrease of the surface area when the radius is 4 cm. **(5 marks)**

[Volume of a sphere is $\frac{4}{3}\pi r^3$ and surface area of a sphere is $4\pi r^2$]

(ii) The finite area enclosed by the line $y = 2 + x$, the $x-$axis and the ordinates $x = 0$ and $x = 3$ is rotated completely about the $x-$axis. Calculate the volume of the solid formed. **(6 marks)**

(iii) Find the position vector of the centre of gravity of particles of masses 2 kg, 1 kg and 3 kg which are at points with position vectors $6\mathbf{i}+6\mathbf{j}$, $3\mathbf{i}+5\mathbf{j}$ and $2\mathbf{i}-\mathbf{j}$ respectively. **(6 marks)**

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11. (i) The forces $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ N, $\mathbf{F}_2 = (-5\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{F}_3 = (4\mathbf{i} + 6\mathbf{j})$ N act on a particle of mass 2 kg. Find;

(a) the resultant of the three forces acting on the particle, **(2 marks)**

(b) the magnitude of the acceleration of the particle. **(4 marks)**

Given that a fourth force, $\mathbf{F}_4 = (x\mathbf{i} + y\mathbf{j})$ N is added to the system to attain equilibrium.

(c) Find the values of x and y. **(3 marks)**

(ii) A particle P is displaced from the origin, O to a point A with position vector $(-3\mathbf{i} + 5\mathbf{j})$ m by a force $(2\mathbf{i} + 7\mathbf{j})$ N. Find the work done by the force in displacing the particle from O to A. **(4 marks)**

(iii) A car is moving at a maximum speed of 10 ms^{-1} on a level road against a constant resistance of 500 N, Find the power generated by the engine. **(4 marks)**

SECTION C: STATISTICS AND PROBABILITY
(IF THIS SECTION IS CHOSEN, THEN SECTION B MAY NOT BE CHOSEN)
IF THIS SECTION IS CHOSEN, THEN ANSWER ANY TWO QUESTIONS

12. The mark distribution of 50 candidates in Physics is given as follows

mark (x)	0 – 6	7 – 13	14 – 20	21 – 27	28 – 34	35 – 41	42 – 48
No. of candidates (f)	4	6	16	10	8	5	1

(a) Draw a histogram to show the distribution above and from it, estimate the mode. **(5 marks)**

(b) Taking 2 cm to represent 5 units on the cumulative frequency axis, and 2 cm to represent 10 units on the mark axis, draw a cumulative frequency graph. **(5 marks)**

(c) Using the graph, find the median mark and the semi-interquartile range. **(3 marks)**

(d) Calculate the mean mark of the distribution. **(4 marks)**

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13. (i) A discrete random variable X has probability mass function defined by

$$f(x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ k(7-x)^2, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

(a) Copy and complete the distribution table below. **(2 marks)**

x	1	2	3	4	5	6
$f(x)$		$4k$			$4k$	

Find;

(b) the value of k. **(3 marks)**

(c) The mean and variance of X **(5 marks)**

(d) $P(X \leq 3)$. **(2 marks)**

(ii) A discrete random variable X is such that $X \sim \text{Bin}(n, 0.3)$ Given that the mean of X is 2.4, find

(e) the value of n, **(2 marks)**

(f) the standard deviation of X. **(3 marks)**

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14. (i) Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{13}{24}$. Calculate

(a) $P(A \cap B)$, **3 marks)**

(b) $P(A' \cap B)$, **3 marks)**

(c) $P(A|B)$. **2 marks)**

(ii) There are two boxes X and Y containing black and red balls. Box X contains 4 black balls and 8 red balls while box Y contains 3 black balls and 5 red balls. A box is chosen at random and a ball drawn from it.

(d) Draw a tree diagram to show all the possible outcomes. **4 marks)**
Hence or otherwise, find the probability that;

(e) a black ball is drawn from box X, **2 marks)**

(f) a red ball is drawn. **3 marks)**
