

GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

0775 FURTHER MATHS 3

JUNE 2023

ADVANCED LEVEL

Subject Title	Further Mathematics
Paper No.	Paper 3
Subject Code No.	0775

Two and a half hours

Answer ALL questions.

For your guidance the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Turn Over

1. A force F_1 of magnitude $2\sqrt{29}$ N acts along the line $\mathbf{r} = [(i + k) + \alpha(3i - 2j - 4k)]\text{m}$

i) Find the work done by F_1 in displacing a particle from the point where $\alpha = 0$ to the point where $\alpha = 2$. (6 marks)

The forces F_2 and F_3 act at points with position vectors $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively, where

$$F_2 = (-i + j)\text{N}, \quad \mathbf{r}_2 = (j + k)\text{m}$$

$$F_3 = (-i + 4k)\text{N}, \quad \mathbf{r}_3 = (i + j + k)\text{m}$$

The system of forces F_1 , F_2 and F_3 is equivalent to a single force, $\mathbf{R}$, acting through the origin O together with a couple $\mathbf{G}$.

Show that

ii) $\mathbf{R} = (4i - 3j - 4k)\text{N}$ (2 marks)

iii) the magnitude of the couple $\mathbf{G}$ is $3\sqrt{13}\text{Nm}$. (6 marks)

2. (a) The coordinates of points on the curve $y = f(x)$ are (x, y) where x and y are given by the table below:

x	0	5	10	15	20	25	30
y	0	9	14	18	22	23	19

Use Simpson's rule with 6 strips to estimate the area enclosed by the curve $y = f(x)$ the x -axis and the ordinates at $x = 0$ and $x = 30$. (4 marks)

(b) Given that

$$\frac{dy}{dx} + x^2 - y^2 = 0 \text{ and that } y = 0 \text{ when } x = 1,$$

find the first three non-zero terms in the Taylor series expansion of y at $x = 1$. (6 marks)

3. A particle P, of mass 2 kg, moves in straight horizontal line with simple harmonic motion with centre O. When P is at a point distant 1.5 metres from O, its speed is 4 metres per second and its acceleration is -6ms^{-2}

Find

- i) the period of motion. (4 marks)
- ii) the amplitude of the motion. (3 marks)
- iii) the maximum speed of P. (2 marks)
- iv) the maximum force acting on P. (3 marks)

4. A car of mass 1500kg starts from rest and moves along a straight level road surface with its engine working at the rate of $6000v$, where v is the velocity of the car at time t . Given that the total resistance to the motion of this car when traveling at speed $v \text{ ms}^{-1}$ is

$$\frac{6000v}{u} \text{ where } u \text{ is a constant,}$$

show that

- i) u is the maximum speed of the car along this road, (3 marks)
- ii) the equation of motion of this car is

$$\frac{4}{u} = \frac{1}{u - v} \frac{dv}{dt}. \quad (2 \text{ marks})$$

Hence, find

- iii) the time taken for the speed of the car to be raised from 0 to $\frac{1}{2}u$. (4 marks)
- iv) the distance travelled by the car in this time. (3 marks)

5. Two smooth spheres P and Q of equal radii and equal masses lie at rest on a smooth horizontal table. Sphere P is given a speed u and it impinges obliquely on Q . The direction of motion of P before impact makes an angle of $\frac{1}{4}\pi$ with the line of centres of the spheres.

i) Find the speeds of P and Q immediately after the collision, leaving your answers in terms of u and e , where e is the coefficient of restitution between the two spheres. (10 marks)

Given that sphere P is deflected by the collision through an angle of $\frac{1}{3}\pi$,

ii) show that $e = 5 - 2\sqrt{3}$. (4 marks)

6. A particle P moves on the curve with polar equation $r = e^\theta$, where (r, θ) are the polar coordinates referred to a fixed pole, in such a way that the transverse component of its acceleration is always zero.

Show that

i) $r^2 \frac{d\theta}{dt} = k$, where k is a constant. (2 marks)

ii) $\frac{dr}{dt} = \frac{k}{r}$. (3 marks)

Hence, when $r = a$,

find

iii) the speed of P . (4 marks)

iv) the radial component of the acceleration of P . (4 marks)

7. A uniform thin rod OA , of mass m and length $6a$ can rotate freely in a fixed vertical plane about a fixed horizontal axis through O . A uniform circular disc of mass $12m$ and radius a is clamped to the rod so that its centre C lies on the rod and its plane coincides with the plane in which the rod can rotate.

When $OC = x$,

i) Find in terms of m , a and x the moment of inertia of the system about the horizontal axis through O . (3 marks)

Show that

ii) the distance of the centre of mass of the composite body from the point O is

$$\frac{1}{13}(3a + 12x). \quad (2 \text{ marks})$$

iii) the period, T , of small oscillations of the system is given by

$$T^2 g(a + 4x) = 8\pi^2(3a^2 + 2x^2). \quad (5 \text{ marks})$$

8. A high-tension electric line passes through a forest. The probability that a tree when cut at random near the line will fall on the line is $\frac{1}{10}$.

Five such trees are cut.

Find, to 4 decimal places, the probability that

i) no trees will fall on the line, (3 marks)

ii) at least one tree will fall on the line, (2 marks)

iii) fewer than three trees will fall on the line. (3 marks)

One hundred of such trees are cut near the line.

Use the Normal approximation, to find to 2 significant figures, the probability that between 6 and 14 trees inclusive will fall on the line. (7 marks)

Go Back and Check Your Work