GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

0775 Further Mathematics 1

JUNE 2023 **ADVANCED LEVEL**

Centre No.	
Centre Name	
Candidate Identification No	
Candidate Name	

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTION PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed "**Advanced Level – 0775 Further Mathematics 1**".
4. Fill in the information required in the spaces above.
5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil:

Candidate Number and Name, Centre Number and Name.

6. Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.

How to answer the questions in this examination

7. Answer **ALL** the **50** questions in this Examination. All questions carry equal marks.
8. Calculators are allowed.
9. Each question has FOUR suggested answers: **A, B, C** and **D**. Decide on which answer is correct. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square bracket for the answer you have chosen.

   For example, if **C** is your correct answer, mark **C** as shown below:

   [A] [B] [~~C~~] [D]

10. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
11. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
12. Do all rough work in this booklet, using, where necessary, the blank spaces in the question booklet.
13. **At the end of the examination, the invigilator shall collect the answer sheet first and then the question booklet after. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

1. $\frac{x^2+1}{(1-x^2)}, x \neq \pm 1$, expressed in partial fractions where $a, b \in \mathbb{R}$ is

A $\frac{a}{1-x}+\frac{b}{1+x}$

B $\frac{a}{1-x}+\frac{b}{1+x}+1$

C $\frac{ax+b}{1-x^2}$

D $\frac{a}{1-x}+\frac{b}{1+x}-1$

2. The integrating factor for the differential equation $x\frac{dy}{dx}-2y=x^2$ is

A $\frac{2}{x}$

B $\frac{1}{x^2}$

C $-\frac{1}{x^2}$

D $-\frac{2}{x^2}$

3. In exponential form, $1-\tanh x \equiv$

A $\frac{2e^{2x}}{e^{2x}+1}$

B $\frac{2}{e^{2x}+1}$

C $\frac{2}{e^{2x}-1}$

D $\frac{2e^{2x}}{e^{2x}-1}$

4. Which one of the following functions has a removable discontinuity?

A $f(x)=\frac{x^2+1}{x+1}$

B $f(x)=\frac{x^2+1}{x-1}$

C $f(x)=\frac{x^2-1}{x-1}$

D $f(x)=\frac{x^2+9}{x-3}$

5. The series $\sum_{r=1}^{\infty}\frac{1}{r^k}$ is convergent for

A $k \geq 1$

B $k > 1$

C $k \leq 1$

D $k < 1$

6. The argument of the complex number $1+e^{-2i\theta}$ is

A θ

B $-\theta$

C 2θ

D -2θ

7. Given the truth table below

p	q	$p \vee q$	$(p \vee q) \Rightarrow q$
T	T	T	T
T	F	T	a
F	T	T	T
F	F	b	F

The truth values of a and b are respectively

A T,T

B T,F

C F,T

D F,F

8. Let $f(x)=[x]$, where $[x]$ is the greatest integer function. $\int_{-2}^{2} f(x)dx =$

A -3

B 0

C 3

D 6

9. The root mean square value of $\sin x$ in the interval $0 \leq x \leq \frac{1}{4}\pi$ is

A $\frac{1}{4}\pi\int_0^{\frac{1}{4}\pi}\sin^2 x dx$

B $\left[\frac{2}{\pi}\int_0^{\frac{1}{4}\pi}\sin^2 x dx\right]^{\frac{1}{2}}$

C $2\left[\frac{1}{\pi}\int_0^{\frac{1}{4}\pi}\sin^2 x dx\right]^{\frac{1}{2}}$

D $\frac{1}{2}\pi\left[\int_0^{\frac{1}{4}\pi}\sin^2 x dx\right]^{\frac{1}{2}}$

10. The Cartesian equation of a curve whose polar equation is $r = 2a\cos\theta$ is

A $x^2 + y^2 - 2ax = 0$
B $x^2 + y^2 - 2ay = 0$
C $x^2 + y^2 + 2ax = 0$
D $x^2 + y^2 + 2ay = 0$

11. The coordinates of the focus of the parabola $y^2 = 16(x+1)$ is

A (4, 0)
B (0, 4)
C (0, 3)
D (3, 0)

12. Given that $(\mathbf{i} - 4\mathbf{k}) \times (2\mathbf{i} - \lambda\mathbf{j} - \mathbf{k}) = -4\mathbf{i} - 7\mathbf{j} - \mathbf{k}$, the value of λ is

A 1
B 0
C -1
D -2

13. A linear transformation, T, is defined as follows

$T : \mathbb{R}^2 \to \mathbb{R}^2$
$(x, y) \mapsto (x - 2y,\ x + y)$

The Kernel of T is:

A $(0,0)$
B $\{(0,0)\}$
C 0
D $\{0,0\}$

14. Given that $(\{a,b,c,d\}, *)$ whose operation table is given below, is a group, then the identity element is

$*$	a	b	c	d
a	b	c	d	a
b	c	d	a	b
c	d	a	b	c
d	a	b	c	d

A a
B b
C c
D d

15. How many distinct solutions are there to the congruence $12x = 15 (\mathrm{mod}\, 21)$?

A 1
B 2
C 3
D 4

16. The asymptotes to the curve $y = f(x)$, where $f(x) = x - 1 + \ln\left|\frac{x-2}{x-1}\right|$, are

A $x = 1,\ x = 2$
B $y = x - 1,\ x = 2$
C $y = x - 1,\ x = 1$
D $y = x - 1,\ x = 2,\ x = 1$

17. The statement $x + 1 = 3$, is true for

A all real values
B at most one value
C at least one value
D just one value

18. A function f is defined by

$$f(x) = \begin{cases} -x, & x < 0 \\ 0, & x = 0 \\ x, & x > 0. \end{cases}$$

Which one of the following statements is true?

A f is odd
B f is **discontinuous** at $x = 0$
C f is an even function
D f is periodic

19. The value of x for which $\sinh^{-1}(2x) = \ln 4$ is:

A $\frac{17}{16}$
B $\frac{15}{8}$
C $\frac{15}{16}$
D $\frac{17}{8}$

20. A similarity transformation (similitude) f, on a complex plane is defined by $z' = (3 - 4i)z + 1 + i$. Its scale factor is

A 1
B $\sqrt{2}$
C 3
D 5

21. $\int_{-1}^{1}\left(x^9 - x^3 + \frac{\tan x}{\left(1+x^4\right)^2}\right)dx =$

A -1
B 0
C 1
D 2

22. The general solution of the differential equation $\frac{d^2x}{dt^2} + 9x = 0$, where p and q are constants is $x =$

A $pe^t\cos(3t+\lambda)$
B $pe^t\sin(3t+\lambda)$
C $p\cos(3t+\lambda)$
D $p\cos(9t+\lambda)$

23. The mean value of $\cosh 2x$ in the interval $0 \le x \le 2$, is

A $\frac{1}{4}\left(e^4 - e^{-4}\right)$
B $\frac{1}{8}\left(e^4 - e^{-4}\right)$
C $\frac{1}{2}\left(e^4 - e^{-4}\right)$
D $8\left(e^4 - e^{-4}\right)$

24. A curve is given in Cartesian form by the equation.
$x^2 + y^2 + 2y = (x^2+y^2)^{\frac{1}{2}}$
The polar equation of the curve is
A $r = 1 - 2\cos\theta$
B $r = 1 - 2\sin\theta$
C $r = 1 + 2\cos\theta$
D $r = 1 + 2\sin\theta$

25. If the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{a^2} = 1$ are perpendicular, then the value of a is

A $\frac{1}{4}$
B $\frac{1}{2}$
C 1
D 4

26. Given that $\mathbf{a}$ and $\mathbf{b}$ are two vectors, which one of the following compound statements is true?
A $\mathbf{a}\bullet\mathbf{b} = \mathbf{0}$ if $\mathbf{a} \perp \mathbf{b}$
B $\mathbf{a}\bullet\mathbf{b} = \mathbf{0}$ if $\mathbf{a} \parallel \mathbf{b}$
C $\mathbf{a}\times\mathbf{b} = \mathbf{c}\times\mathbf{d} \Rightarrow \mathbf{a} = \mathbf{c}$ and $\mathbf{b} = \mathbf{d}$
D $\mathbf{a}\times\mathbf{b} \parallel \mathbf{a}$ if $\mathbf{a} \perp \mathbf{b}$

27. For what value of p is the matrix

$\begin{pmatrix} 4 & 3 & 3 \\ p & 5 & 6 \\ 6 & 7 & 9 \end{pmatrix}$ non-invertible?

A 5
B 6
C 7
D 9

28. A necessary and sufficient condition to show that a group is **NOT** Abelian is
A Closure
B Existence of identity element
C Associativity
D Commutativity

29. The solution of the system of congruences
$x \equiv 1 (\mathrm{mod}\, 2)$
$x \equiv 2 (\mathrm{mod}\, 3)$ is
A $x \equiv 2(\mathrm{mod}\, 6)$
B $x \equiv 3(\mathrm{mod}\, 6)$
C $x \equiv 4(\mathrm{mod}\, 6)$
D $x \equiv 5(\mathrm{mod}\, 6)$

30. The solutions of $z^3 = -8$ are $z =$
A $\{-2, 1 \pm i\sqrt{3}\}$
B $\{-2, -2 \pm i\sqrt{3}\}$
C $\{-2, 2 \pm i\sqrt{3}\}$
D $\{2, -1 \pm i\sqrt{3}\}$

31. The general solution to the differential equation $4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$ for constants A, B is $y =$

A $(A + Bx)e^{\frac{3}{4}x}$
B $A + Be^{\frac{3}{4}x}$
C $A + Bxe^{\frac{3}{4}x}$
D $A + Be^{\frac{4}{3}x}$

32. $\int \frac{1}{\sqrt{9+x^2}}dx =$, where k is a constant is,

A $\sin^{-1}\left(\frac{x}{3}\right) + k$
B $\cos^{-1}\left(\frac{x}{3}\right) + k$
C $\sinh^{-1}\left(\frac{x}{3}\right) + k$
D $\cosh^{-1}\left(\frac{x}{3}\right) + k$

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33. The equation $\frac{x^2}{2-a} + \frac{y^2}{a-5} = -1$ represents **an** hyperbola if

A $a > 2$
B $a < 5$
C $2 < a < 5$
D $a > 5$

34. Given that $M = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ are elements of S_3, the symmetric group of permutations of $A = \{1,2,3\}$, the composition $M \circ N =$

A $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$
B $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$
C $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$
D $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

35. If the rate at which cells in a certain organism die is inversely proportional to the number n, of cells present at time t. Treat n as a continuous variable, with k, a positive constant. Then, which one of the differential equations is satisfied by n?

A $\frac{dn}{dt} = -\frac{k}{n}$
B $\frac{dn}{dt} = -\frac{1}{n}$
C $\frac{dn}{dt} = \frac{1}{n}$
D $\frac{dn}{dt} = \frac{k}{n}$

36. Using the approximation $y_{n+1} \cong y_n + 2h\left(\frac{dy}{dx}\right)_n$ and a step length of 0.1, the approximate value of y, when $x = 1.1$, given that $\frac{dy}{dx} = x + y$ and $y = 0$ when $x = 1$ is:

A 1.2
B 1.68
C 1.4
D 0.2

37. A particle performing SHM has a speed of $v\,ms^{-1}$ when it is 1 m from the centre of oscillations. If the period of oscillations is $\sqrt{2}\pi s$, and the amplitude 3 m, then value of v in ms^{-1} is

A 1
B 2
C 3
D 4

38. The moment of inertia of a uniform rod of mass m and length $3l$ about an axis through its centre and perpendicular to the rod is

A $\frac{1}{4}ml^2$
B $\frac{1}{3}ml^2$
C $\frac{3}{4}ml^2$
D ml^2

39 A force $\mathbf{F} = (2\mathbf{i} - \mathbf{j} + \mathbf{k})N$ acts on particle giving it a displacement of $(\mathbf{i} + \mathbf{j} + 2\mathbf{k})m$. The work done by $\mathbf{F}$ is

A $3\,N$
B $\sqrt{27}\,N$
C $-6\,N$
D $9\,N$

40. A particle P moves with constant angular speed ω on a curve whose polar equation is $r = (1 - \cos\theta)$. The traverse component of its acceleration is

A $-2\omega \sin\theta$
B $2\omega \sin\theta$
C $2\omega^2 \sin\theta$
D $-2\omega^2 \sin\theta$

41. The coefficient of restitution e of two colliding objects satisfies the inequality

A $0 \leq e \leq 1$
B $0 \leq e < 1$
C $0 < e \leq 1$
D $0 < e < 1$

42. The value of k for which

$$p(x) = \begin{cases} k(x+2), & 0 \leq x \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

is a probability density function is

A $\frac{1}{24}$

B $\frac{1}{16}$

C $\frac{1}{10}$

D $\frac{3}{110}$

43. A liquid is filling a container at the rate of $\frac{2}{x}$ litres per second where x is the quantity in litres, of liquid in the container after time t seconds. 10 litres are initially found in the container. How many litres will be found in the container after 75 seconds?

A 70
B 50
C 20
D 10

44. Given that $\frac{dy}{dx} = x - y$ and $y = 1$ when $x = 0$, the value of y when $x = 0.1$, using a step length of 0.1 and the approximation $y_{n+1} \cong y_n + 2h\left(\frac{dy}{dx}\right)_n$ is

A 0.669
B 0.786
C 0.80
D 0.90

45. A particle oscillates through a distance $\frac{4}{5}m$ on either side of its equilibrium position with period of 4 seconds. The maximum speed of the particle in ms^{-1} is

A $\frac{1}{5}\pi$

B $\frac{2}{5}\pi$

C $\frac{16}{5}\pi$

D 32π

46. If the moment of inertia of a square lamina of side a and mass m about an axis through its centre and in its plane is I, then its moment of inertia about an axis through the midpoint of one edge and perpendicular to the plane of the lamina is

A $I + \frac{1}{2}ma^2$

B $2I + \frac{1}{2}ma^2$

C $I + \frac{1}{4}ma^2$

D $2I + \frac{1}{4}ma^2$

47. If the force $\mathbf{F} = (5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})N$ acts through the point with position vector $\mathbf{r} = (2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})m$, then the magnitude of its moment in Nm about $\mathbf{O}$ is

A 29.1
B 29
C 28.3
D 25

48. A particle is moving round the polar curve $r = a\sin\theta$ with constant angular velocity ω. The transverse component of its velocity when $\theta = \frac{\pi}{6}$ is

A $\frac{1}{2}a\omega$

B $\frac{\sqrt{3}}{2}a\omega$

C $a\omega$

D 0

49. A smooth sphere moving in space impinges on a similar sphere such that its velocity changes from $(2\mathbf{i} - \mathbf{j} - 3\mathbf{k})ms^{-1}$ to $(3\mathbf{i} + \mathbf{j} - \mathbf{k})ms^{-1}$. The direction of the line of centres is

A $(5\mathbf{i} - 4\mathbf{k})$
B $(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$
C $(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$
D $(3\mathbf{i} + \mathbf{j} - \mathbf{k})$

50. The equivalent of $P(|z| < a)$ is

A $2\Phi(a)$
B $2(1 - \Phi(a))$
C $2\Phi(a) - 1$
D $1 - \Phi(a)$

STOP
GO BACK AND CHECK YOUR WORK