

GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

0765 Pure Maths With Mechanics 1

JUNE 2023

ADVANCED LEVEL

Centre No.	
Centre Name	
Candidate Identification No	
Candidate Name	

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTIONS PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed "Advanced Level– 0765 Pure Maths With Mechanics 1".
4. Fill in the information required in the spaces above.
5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil:
Candidate Number and Name, Centre Number and Name.
Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.

6. **Answer All questions.**

7. **Formulae Booklets and calculators are allowed.**

8. Each question has FOUR suggested answers: A, B, C and D. Decide on which answer is correct. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square brackets for the answer you have chosen.

For example, if C is your correct answer, mark C as shown below:

[A] [B] ☒ [C] [D]

9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet, using, where necessary, the blank spaces in the question booklet.
12. **At the end of the examination, the invigilator shall collect the answer sheet first and then the question booklet after. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

Turn over

SECTION A: PURE MATHEMATICS

1. If $\frac{x}{(x+3)(x+4)} \equiv \frac{P}{x+3} + \frac{Q}{x+4}$, then

- A $P = -3, Q = 4$
- B $P = 4, Q = -3$
- C $P = 3, Q = -4$
- D $P = 3, Q = 4$

2. The quadratic equation $\lambda x^2 - 4x + \lambda = 0$ has real and distinct roots only when

- A $16 - 4\lambda^2 < 0$
- B $16 - 4\lambda^2 > 0$
- C $16 - 4\lambda^2 \leq 0$
- D $16 - 4\lambda^2 \geq 0$

3. An equivalence relation, $\mathcal{R}$, is that which is

- A reflexive, symmetric and anti-symmetric
- B transitive, reflexive and symmetric
- C reflexive, injective and symmetric
- D injective, surjective and bijective

4. When the polynomial $P(x) = ax^3 - x^2 - x + 6$ is divided by $(x + 1)$, the remainder is 4. The value of the constant a is

- A -2
- B 2
- C -4
- D 4

5. The value of $\frac{1}{3} \log_2 \sqrt{8}$ is

- A $\frac{1}{2}$
- B $\frac{1}{3}$
- C $\frac{3}{5}$
- D $\frac{5}{2}$

6. The sum of the first n terms of a series is given by $S_n = n^2 + 5$. The 5th term of this series is

- A 30
- B 18
- C 9
- D 51

7.

$$\sum_{r=1}^4 \left(4^{\frac{1}{2}}\right)^r =$$

- A 16
- B 30
- C 32
- D 64

8. Two real valued functions f and g are defined by

$$f(x) = \frac{2x-3}{x+1}, x \neq -1 \text{ and}$$

$$g(x) = \frac{1}{x}, x \neq 0, (g \circ f)(0) =$$

- A ∞
- B 0
- C $-\frac{1}{3}$
- D $\frac{1}{3}$

9. The general solution of the differential equation

$$2y \frac{dy}{dx} = 1 + x \text{ is}$$

- A $2y = x + \frac{x^2}{2} + k$
- B $2y = x - \frac{x^2}{2} + k$
- C $y^2 = x - \frac{x^2}{2} + k$
- D $y^2 = x + \frac{x^2}{2} + k$

10. The modulus of the complex number $\frac{1+2i}{3-4i}$ is

- A $\frac{\sqrt{4}}{5}$
 B $\frac{\sqrt{5}}{3}$
 C $\frac{\sqrt{5}}{4}$
 D $\frac{\sqrt{5}}{5}$

11. If $(1+2x)^{-1} \equiv 1-2x+bx^2$, then the value of b is

- A -2
 B 2
 C -4
 D 4

12. The general solution of the equation $\sin \theta - \cos \theta = 0$ is

- A $n\pi + \frac{\pi}{4}$
 B $n\pi - \frac{\pi}{4}$
 C $2n\pi \pm \frac{\pi}{4}$
 D $2n\pi + \frac{\pi}{4}$

13. The sine of the acute angle between the plane Π and the line L , where

$$\Pi: x + y + z = 2$$

$$L: \frac{x-3}{2} = \frac{y+5}{2} = \frac{z-2}{1} \text{ is}$$

- A $\frac{-2}{3\sqrt{8}}$
 B $\frac{2}{\sqrt{38}}$
 C $\frac{3\sqrt{3}}{5}$
 D $\frac{\sqrt{3}}{5}$

14. If n is a positive integer, then

$$\lim_{n \rightarrow \infty} \left[\frac{n+3}{2n+1} \right] =$$

- A $\frac{1}{2}$
 B 2
 C -3
 D 3

15. $\frac{\sin 5x - \sin x}{\sin 4x - \sin 2x} \equiv$

- A $2 \cos 2x$
 B $2 \sin x$
 C $2 \cos x$
 D $2 \sec x$

16. The cofactor of the element 6 in the matrix

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \text{ is}$$

- A $\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$
 B $-\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$
 C $\begin{vmatrix} 8 & 7 \\ 2 & 1 \end{vmatrix}$
 D $-\begin{vmatrix} 7 & 8 \\ 1 & 2 \end{vmatrix}$

17. Two statements p and q are given by

p : James is dancing

q : James is awake.

Then $\sim(p \wedge q)$ means

- A James is not dancing and James is not awake
 B James is not dancing or James is not awake
 C James is not dancing and James is awake
 D James is not dancing or James is awake

18. The diameter of a circle is the line segment AB . If the coordinates of A and B are $(1, 8)$ and $(3, 14)$ respectively, then the equation of the circle is

- A $(x-14)(x-8) + (y-3)(y-1) = 0$
 B $(x-3)(x-14) + (y-1)(y-8) = 0$
 C $(x-1)(x-14) + (y-3)(y-8) = 0$
 D $(x-1)(x-3) + (y-8)(y-14) = 0$

Turn Over

19. If $y^2 - y + 2x^2 = 5$, then $\frac{dy}{dx} =$

- A $\frac{4x}{1-2y}$
 B $\frac{4x}{2y-1}$
 C $\frac{4x}{1-y}$
 D $\frac{4x}{y-1}$

20. The image of the point $(1, 0, 3)$ under the transformation whose matrix is M where

$$M = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & 3 & 1 \end{pmatrix} \text{ is the point}$$

- A $(1, 3, 3)$
 B $(0, 3, 3)$
 C $(1, 0, 0)$
 D $(3, 0, 3)$

21. If the vectors

$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{s} = 5\mathbf{i} - a\mathbf{j} + 2\mathbf{k}$ are perpendicular then the value of a is

- A -12
 B -8
 C 8
 D 12

22. Given that $y = x^2 - \frac{1}{x}$, $\frac{dy}{dx} =$

- A $2x - \frac{1}{x^2}$
 B $2x + 1$
 C $2x + \frac{1}{x^2}$
 D $2x - 1$

23. $\int \frac{x}{1+3x^2} dx =$

- A $6 \ln(1 + 3x^2) + k$
 B $\frac{1}{6} \ln(1 + 3x^2) + k$
 C $\frac{1}{3} \ln(1 + 3x^2) + k$
 D $3 \ln(1 + 3x^2) + k$

24. A real valued function f is defined by

$$f(x) = \frac{1}{x-2}, x \neq 2.$$

The range of f is

- A $\{x \in \mathbb{R} : x \neq 2\}$
 B $\{x \in \mathbb{R} : x \neq -2\}$
 C $\{x \in \mathbb{R} : x \neq 0\}$
 D $\{x \in \mathbb{R} : x \neq 1\}$

25. The range of values of x for which

$$\frac{x+5}{x-3} < 0 \text{ is}$$

- A $x < -5$ or $x > 3$
 B $x < 3$ or $x > 5$
 C $-5 < x < 3$
 D $3 < x < 5$

26. The complex number i^{102} is equal to

- A 1
 B -1
 C i
 D $-i$

27. Given that -1.5 is the first approximations to the root of the equation $f(x) = 0$, where $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function, a second approximation of the root of the equation is

A	$-1.5 - \frac{f'(-1.5)}{f(-1.5)}$
B	$-1.5 + \frac{f(-1.5)}{f'(-1.5)}$
C	$-1.5 - \frac{f(-1.5)}{f'(-1.5)}$
D	$-1.5 + \frac{f'(-1.5)}{f(-1.5)}$

28. If the set $B = \{4, 5, 8, 10\}$, the cardinality of the power set of B is

- A 4
 B 14
 C 16
 D 8

29. $\int_0^{\frac{\pi}{2}} 6 \cos 2x \, dx =$

- A 3
B 0
C 2
D 1

30. The parametric equations of a curve are
 $x = 3t + \sin 3t$ and $y = 3 - \cos 3t$. $\frac{dy}{dx} =$

- A $\frac{\sin 3t}{3 + \cos 3t}$
B $\frac{3 + \cos 3t}{\sin 3t}$
C $\frac{1 + 3\cos 3t}{\sin 3t}$
D $\frac{1 + \cos 3t}{3 + \cos t}$

31. The equations of the three asymptotes of the curve

$$y = \frac{x-2}{(x+5)(x-1)} \text{ are}$$

- A $y = 1, x = 5, x = 2$
B $y = 0, x = -5, x = 1$
C $y = 0, x = 5, x = 1$
D $y = 1, x = -5, x = 1$

32. $\frac{\cos 5\theta - i \sin 5\theta}{\cos 3\theta - i \sin 3\theta} =$

- A $\cos 2\theta - i \sin 2\theta$
B $\cos 2\theta + i \sin 2\theta$
C $\cos 8\theta - i \sin 8\theta$
D $\cos 8\theta + i \sin 8\theta$

33. A committee of 6 is to be formed from 10 men and 6 women. In how many ways can this selection be done such that there should be an equal number of men and women?

- A ${}^{10}C_3 + {}^6C_3$
B ${}^{10}C_6 + {}^6C_6$
C ${}^3C_{10} + {}^3C_6$
D ${}^{10}C_3 \cdot {}^6C_3$

34. The table below gives values of y corresponding to some given values of x .

Using the trapezium rule, an approximate value for

$$\int_1^5 y \, dx \text{ is.}$$

- A 32
B 36
C 16
D 28

35. $\int \sin 8x \cos 2x \, dx =$

A	$-\left(\frac{\cos 10x}{10} + \frac{\cos 6x}{6}\right) + K$
B	$\left(\frac{\cos 10x}{10} + \frac{\cos 6x}{6}\right) + K$
C	$-\frac{1}{2}\left(\frac{\cos 10x}{10} + \frac{\cos 6x}{6}\right) + K$
D	$\frac{1}{2}\left(\frac{\cos 10x}{10} + \frac{\cos 6x}{6}\right) + K$

SECTION B: MECHANICS

36. The position vector of a particle is
 $\mathbf{r} = [(t^3 + 2t^2 + 2)\mathbf{i} + (t^2 - 1)\mathbf{j}]$ m at time t s.
The acceleration of the particle when $t = 1$ is

- A $(10\mathbf{i} - 2\mathbf{j})\text{ms}^{-2}$
B $(2\mathbf{i} + 10\mathbf{j})\text{ms}^{-2}$
C $(7\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$
D $(10\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$

37. A particle is projected from a point on a horizontal plane with speed 10 m s^{-1} at an angle 30° to the plane. Taking g as 10 m s^{-2} , the greatest height H attained by the particle is

- A $H = 5 \text{ m}$
B $H = 1.25 \text{ m}$
C $H = \frac{5}{2}\sqrt{3} \text{ m}$
D $H = 5\sqrt{3} \text{ m}$

38. The work that must be done to compress a spring of modulus 500 N from its natural length of 0.1 m to a shortened length of 0.08 m is
- A 1 J
B 0.1 J
C 2 J
D 50 J

39. A particle of mass 4 kg is moving in a horizontal circle of radius 2 m with angular speed of 3 rad s^{-1} . The centripetal force acting on the particle is
- A 18 N
B 72 N
C 24 N
D 3 N

40. Three particles of weight 7 N, 9 N and 4 N have position vectors $(4\mathbf{i} + \mathbf{j}) \text{ m}$, $3\mathbf{j} \text{ m}$ and $(3\mathbf{i} + 4\mathbf{j}) \text{ m}$ respectively. The position vector of their center of mass is

- A $\left(\frac{7}{3}\mathbf{i} + \frac{8}{3}\mathbf{j}\right) \text{ m}$
B $\left(2\mathbf{i} + \frac{5}{2}\mathbf{j}\right) \text{ m}$
C $(40\mathbf{i} + 50\mathbf{j}) \text{ m}$
D $\left(\frac{7}{20}\mathbf{i} + \frac{8}{20}\mathbf{j}\right) \text{ m}$

41. Fig. 1 shows a uniform sheet of metal in the form of the letter T with dimensions in meters.

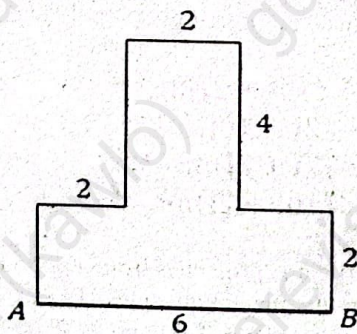


Fig. 1

The distance of the center of mass of the sheet from the edge AB is

- A 2.5 m
B 1.3 m
C 2.2 m
D 3 m

42. A uniform ladder AB of length $2a$ rests with its upper end B against a rough vertical wall and the lower end A against a rough horizontal floor. The forces acting on the ladder AB are shown in fig. 2.

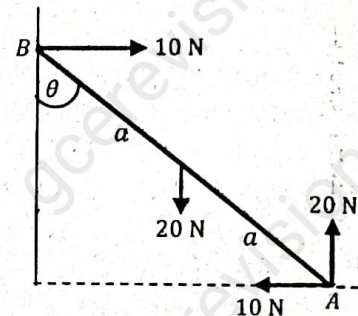


Fig. 2

Given that AB is in limiting equilibrium, the angle θ which AB makes with the wall is

- A 35°
B 30°
C 60°
D 45°

43. X, Y and Z are moving particles. The velocity of X is $(4\mathbf{i} + 5\mathbf{j}) \text{ ms}^{-1}$. Relative to X the velocity of Y is $(3\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ and relative to Y the velocity of Z is $(5\mathbf{i} - 3\mathbf{j}) \text{ ms}^{-1}$. The true velocity of Z is

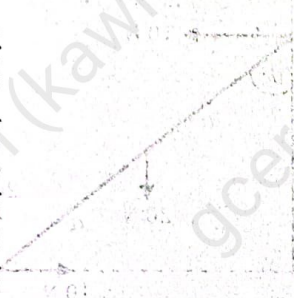
- A $(12\mathbf{i} + 4\mathbf{j}) \text{ ms}^{-1}$
B $(9\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$
C $(7\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$
D $(8\mathbf{i} - \mathbf{j}) \text{ ms}^{-1}$

44. A particle moves so that after t seconds its position vector $\mathbf{r} = [(4t - 1)\mathbf{i} + t^2\mathbf{j} + (15t - t^3)\mathbf{k}] \text{ m}$. The speed of the particle after 3 seconds is

- A $(4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}) \text{ ms}^{-1}$
B $(3\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}) \text{ ms}^{-1}$
C 14 ms^{-1}
D $\sqrt{189} \text{ ms}^{-1}$

45. A sphere of mass 2 kg collides directly with another sphere of mass 8 kg. Both spheres are moving in the same direction with speeds 3 ms^{-1} and 1 ms^{-1} respectively. Given that the 2 kg mass is brought to rest by the impact, the coefficient of restitution between the spheres is

A $\frac{1}{2}$
 B $\frac{7}{8}$
 C $\frac{4}{7}$
 D $\frac{3}{7}$



46. The work done by a force $\mathbf{F} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})\text{N}$ which moves its point of application from a point P with position vector $\mathbf{r}_P = (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})\text{m}$ to another point Q with position vector $\mathbf{r}_Q = (7\mathbf{i} + 6\mathbf{j} + 5\mathbf{k})\text{m}$ is

A -60 J
 B 60 J
 C -8 J
 D 8 J

47. A car of mass 1200 kg moving with a speed of 12 ms^{-1} has its engine working at 54 kW up a road inclined at an angle $\sin^{-1}(\frac{1}{5})$ to the horizontal. The non-gravitational resistance to the motion of the car is 300 N. Taking g as 10 ms^{-2} , the acceleration of the car is

A 2.5 ms^{-2}
 B 1.5 ms^{-2}
 C 3.5 ms^{-2}
 D 1.2 ms^{-2}

48. Taking g as 10 ms^{-2} , the work done in lifting 8 blocks, each of weight 12 kg, from the floor onto a shelf 2.8 m high is

A 2688 J
 B 268.8 J
 C 26.88 J
 D 2.688 J

49. A car accelerates uniformly from a velocity of 10 ms^{-1} to 60 ms^{-1} in 6 seconds on a straight level road. The distance covered by the car is

A 150 m
 B 200 m
 C 210 m
 D 215 m

50. Given that the events A and B are such that $P(A|B) = \frac{11}{21}$ and $P(B) = \frac{3}{5}$, then $P(A \cap B) =$

A $\frac{5}{7}$
 B $\frac{4}{7}$
 C $\frac{3}{7}$
 D $\frac{2}{7}$

STOP
GO BACK AND CHECK YOUR WORK