GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

Pure Maths With Mechs 2
0765/2

JUNE 2023 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Mechanics**
Paper No.	**Paper 2**
Subject Code No.	**0765**

Three Hours.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the GCE Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page

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1. (i) Given that $(x-1)$ and $(x+1)$ are factors of the polynomial $f(x) = ax^4 + 7x^3 + x^2 + bx - 3$, find the values of the constants a and b. **(3 marks)**
Hence, find the set of real values of x for which $f(x) > 0$. **(4 marks)**

(ii) Given that $m\log_3 2 = 10\log_9 6$. Find the values of m. **(3 marks)**

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2. The table below shows the values of two continuous variables x and y obtained experimentally.

x	2	3	4	5	6
y	1	0.75	0.67	0.63	0.60

It is known that x and y have a relation of the form $\frac{a}{x} + \frac{b}{y} = 1$.
By drawing the graph of $\frac{1}{y}$ against $\frac{1}{x}$, estimate, to one decimal place, the values of the conatants a and b. **(8 marks)**

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3. (i) Given that $z_1 = 1 + \sqrt{3}i$ and $z_2 = -1 - \mathrm{i}$,
(a) express z_1 and z_2 in the form $r(\cos\theta + i\sin\theta)$, where $r \in \mathbb{R}$ and $-\pi < \theta \leq \pi$.

(b) evaluate $\left|\frac{z_1}{z_2}\right|^3$ and $\arg\left(\frac{z_1}{z_2}\right)^3$. **(7 marks)**

(ii) Prove by mathematical induction that $9^n - 1$ is divisible by 8, $\forall\, n \in \mathbb{N}$. **(5 marks)**

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4. (i) Given that $f(x) = \sin x \ln(\tan x)$, find $f'(x)$ when $x = \frac{\pi}{4}$. **(5 marks)**

(ii) Evaluate $\int_2^{10} \frac{x}{\sqrt{x-1}}\, dx$, using the substitution $u^2 = x - 1$. **(5 marks)**

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5. (i) Solve the equation $\cos x = \sin 2x$ in the interval $0 \leq x \leq \pi$. **(5 marks)**

(ii) Express $5\cos x - 12\sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$, and $0^\circ < \alpha < 90^\circ$, giving the value of α correct to one decimal place.
Hence or otherwise, find the minimum and maximum values of
$$\frac{10}{|5\cos x - 12\sin x| + 15}.$$ **(6 marks)**

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6. (i) Solve the differential equation $\frac{dy}{dx} = xe^{2y}$, given that $y = 0$ when $x = 0$, expressing the answer in the form $y = f(x)$. **(5 marks)**

(ii) A relation R defined on the set $S = \{1,\ 2,\ 3,\ 4\}$ is given by
$R = \{(1,1), (1,3), (2,2), (3,1), (3,3), (4,4)\}$.

(a) Show that R is an equivalence relation. **(3 marks)**
(b) Write down the equivalent classes of the elements 1 and 4 in R. **(2 marks)**

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7. (i) A function f: $\mathbb{R} - \{-3\} \longrightarrow \mathbb{R} - \{1\}$ is defined by $f(x) = \frac{x}{(x+3)}$,

Show that f is bijective. **(5 marks)**

(ii) Given the periodic function f with period 6, where

$$f(x) = \begin{cases} x-2, & 2 \leq x \leq 5, \\ -\frac{1}{2}x+4, & 5 < x \leq 8, \end{cases}$$

(a) show that f is not continuous at $x = 5$. **(2 marks)**

(b) find f(19). **(2 marks)**

(c) sketch the graph of $y = f(x)$ in the interval $-1 \leq x \leq 8$. **(3 marks)**

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8. (i) Show that a root of the equation $x^3 + x^2 + 3x - 2 = 0$ lies in the interval $-1 < x < 1$. **(3 marks)**

(ii) A line L and a plane π have equations

L: $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$,

π: $x - 2y + 2z = 3$.

(a) Show that the point $P(2, -2, 3)$ lies on L. **(2 marks)**

(b) Find the position vector of the point of intersection of L and π. **(4 marks)**

(c) Determine the sine of the angle between L and π. **(3 marks)**

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9. (i) Given that $\left(\frac{x}{2x-1}\right) + \left(\frac{x}{2x-1}\right)^2 + \left(\frac{x}{2x-1}\right)^3 + \cdots = \frac{3}{4}$, find the value of x. **(3 marks)**

(ii) A linear transformation T is represented by the matrix $\mathbf{A}$, where $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.

Find

(a) the image of the point $(-5, 2, 1)$ under T. **(2 marks)**

Given that $\mathbf{B}$ is another matrix, where $\mathbf{B} = \begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$,

(b) find the matrix product $\mathbf{BA}$.

Hence or otherwise find the point where the three planes with equations

$x + 2y + 3z = 5$

$2x + 4y + 5z = 9$

$3x + 4y + 6z = 12$, intersect. **(2, 5 marks)**

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10. (i) Show that the expression $x^2 + 2x + 6$ is positive for all real values of x.

Hence, find the set of values of x for which $\frac{x}{x-2} > \frac{3}{x+5}$. **(5 marks)**

(ii) Three cards are to be drawn from a pack of 52 playing cards. In how many ways can this be done if

(a) the cards must consists of 2 clubs and 1 diamond, **(2 marks)**

(b) two or more cards must not come from the same suit. **(3 marks)**
