GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

Pure Maths With Mechs 3
0765/3

JUNE 2023 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics with Mechanics**
Paper No.	**Paper 3**
Subject Code No.	**0765**

Three hours

Full marks may be obtained for answers to ALL questions.

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae booklet published by the GCE Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. A particle P of mass 3 kg moves so that its position vector $\mathbf{r}$ at time t seconds is given by
$$\mathbf{r} = [t\,(1-t)\mathbf{i} + t(\sin \pi t)\mathbf{j}] \text{ m}.$$
Find, when $t = 1$,
(a) the momentum of P, **(5 marks)**
(b) the kinetic energy of P, **(3 marks)**
(c) the cosine of the angle between the velocity and the acceleration. **(5 marks)**

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2. Two particles of mass 6 kg and M kg are connected by a light inelastic string passing over a fixed smooth pulley and then released from rest. The acceleration of the system is 5 m s^{-2}.
Find,
(a) the value of M, **(5 marks)**
(b) the tension in the string, **(2 marks)**
(c) the force exerted by the string on the pulley, **(2 marks)**
(d) the velocity after 2 seconds, **(2 marks)**
(e) the distance each mass travels in 2 seconds. **(2 marks)**

(Take g as 10 m s^{-2}.)

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3. Two smooth spheres A and B of mass $3m$ and $2m$ respectively lie in a straight line, on a smooth horizontal floor, with sphere B between A and a smooth vertical wall. A is projected with speed $2u$ to collide directly with B, and B in turn strikes the wall. The coefficient of restitution of either impact is $\frac{1}{4}$. Find
(a) the speeds of A and B after the first collision, **(5 marks)**
(b) the kinetic energy loss in the first impact. **(2 marks)**
After B rebounds from the wall, show that the speed of sphere A after the second collision with B is $\frac{5u}{16} \text{ m s}^{-1}$. **(6 marks)**

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4. A lorry of mass 1400 kg moving along a straight horizontal road at a steady speed of 20 m s^{-1} with its engine working at the rate of 96 kW experiences constant a non-gravitational resistance R.
(a) Find R. **(3 marks)**
The lorry descends a plane inclined at $\sin^{-1}\left(\frac{1}{10}\right)$ to the horizontal with a steady speed of 12 m s^{-1}.
Given that the non-gravitational resistance remains unchanged,
(b) calculate the rate at which the engine is now working, **(5 marks)**
(c) find the retardation of the lorry if the rate of working of the engine is decreased by 20 kW. **(5 marks)**

(Take g as 10 m s^{-2}.)

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5. (i) A particle is projected from a point A on a horizontal with speed 50 m s^{-1} at an angle of elevation α so that it passes through the point $(120, 10)$.
(a) Show that there are two possible angles of projection. **(5 marks)**
If the angle of projection is 30^o, find,
(b) the maximum height, **(2 marks)**
(c) the range of the particle on the horizontal plane through the point of projection, **(2 marks)**
(d) the time of flight. **(2 marks)**

(ii) The velocity of a particle A relative to particle B is $(2\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$ and the velocity of particle C relative to A is $(5\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$. Find the velocity of C relative to B. **(2 marks)**

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6. (i) A particle of mass 8 kg falls freely under gravity through a medium whose resistance to motion is kv, where v is the speed of the particle and k is a positive contant.

(a) Prove that after time t seconds $\frac{dv}{dt} = \frac{80-kv}{8}$. **(4 marks)**

(b) Hence, show that the speed of the particle at time t seconds is $\frac{80}{k}(1 - e^{-kt})$. **(6 marks)**

(ii) Forces $\mathbf{F}_1 = (7\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{F}_2 = (\mathbf{i} - 5\mathbf{j})$ N act at points with position vectors $\mathbf{r}_1 = (4\mathbf{i} - 5\mathbf{j})$ m and $\mathbf{r}_2 = (3\mathbf{i} - 8\mathbf{j})$ m respectively.
Find the magnitude of the resultant moment. **(3 marks)**

(Take g as 10 m s^{-2}.)

7.

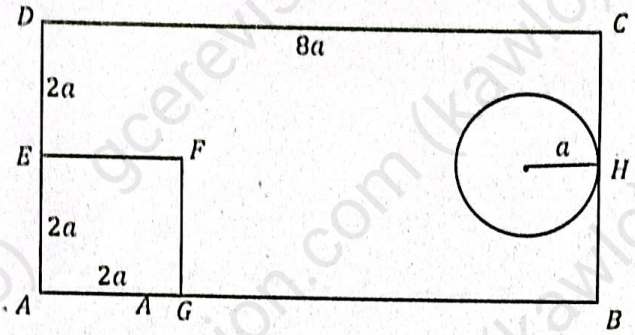

Fig. 1

Fig. 1 shows a uniform rectangular lamina $ABCD$ of sides $8a$ and $4a$. A square $AGFE$ of side $2a$ and a circle of diameter $2a$ with BC as tangent at its midpoint H are removed from the rectangular lamina.
Find,

(a) the distance from AB and AD of the centroid of the remainder of the lamina, **(10 marks)**

(b) show that the tangent of the angle of inclination of AD to the vertical when the remainder of the lamina is suspended freely from D in equilibrium is $\frac{(124-7\pi)a}{52-2\pi}$. **(3 marks)**

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8 (i) Two events A and B are such that $P(A) = \frac{2}{5}$, $P(A \cup B) = \frac{7}{10}$.
Find

(a) $P(B)$, if A and B are mutually exclusive, **(2 marks)**

(b) $P(B)$, if A and B are independent. **(3 marks)**

(ii) When a contractor needs a truck, it could be hired from one of three Companies X, Y or Z.
Of the trucks hired 40% are from X, 50% from Y and 10% from Z.
9 % of trucks hired from X, 6 % of trucks hired from Y and 20 % of trucks hired from Z all arrived late at the contractor's estate.
A truck is hired at random from a company. Calculate the probability that the truck

(c) will arrive late, **(3 marks)**

(d) will not arrive late, **(2 marks)**

Given that a truck hired arrives late, find the probability that it comes from Y. **(3 marks)**

GO BACK AND CHECK YOUR WORK