GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

ADDITONAL MATHEMATICS 2
0575

JUNE 2023   **ORDINARY LEVEL**

Subject Title	Additional Mathematics
Paper No.	2
Subject Code No.	0575

Two and a half hours

Answer ALL QUESTIONS IN SECTION A and ANY TWO QUESTIONS FROM EITHER SECTION B or SECTION C. IN SECTIONS B AND C, ALL QUESTIONS CARRY EQUAL MARKS.

Candidates are expected to answer a combination of Section A **and** Section B **OR** Section A **and** Section C but **NOT** a combination of all three

All necessary working must be shown. No marks will be awarded for answers without brief statements showing how the answers have been obtained.

Electronic calculators are allowed.

Formulae booklets are allowed.

Where necessary take g as 10ms^{-2}.

Turn Over

SECTION A: PURE MATHEMATICS
THIS SECTION IS COMPULSORY TO ALL CANDIDATES
(ANSWER ALL QUESTIONS)

1. (i) Given that $(x - 1)$ is a factor of $f(x)$, where $f(x) = x^3 - 2x^2 - kx + 6$.

    a) Find the value of k,   **(2 marks)**
        With this value of k,
    b) Factorise $f(x)$ completely.   **(2 marks)**

  (ii) Given that α and β are the roots of the equation $2x^2 - x - 1 = 0$.

    a) Find the values of $\alpha + \beta$ and $\alpha\beta$ **(1 mark)**
    b) Hence, find a quadratic equation with integral coefficient whose roots
       are α^2 and β^2.   **(3 marks)**

---

2. (i) In how many ways can 3 boys and 2 girls be chosen from a class of 6 boys and 4 girls.   **(3 marks)**
  (ii) Expand $(1 - 2x)^{-1}$ in ascending powers of x, up to and including the term in x^3 **(4 marks)**

---

3. Chia, Mbako and Fru started a business with total amount of 350,000 FCFA. Of this amount, Chia contributed 50,000 FCFA, Mbako contributed 100,000 FCFA and Fru contributed 200,000 FCFA. The monthly profit they made was shared amongst the three of them in the ratio of the amount they contributed. Given that they made a total profit of 21,000 FCFA in the first month, 28,000 FCFA in the second month and 35,000 FCFA in the third month and that the profit kept increasing in this manner for the first 12 months,

(a) State the ratio of their shares in the simplest form.   **(2 marks)**
    Study the table below and answer the questions that follow:

Month	Total profit made in FCFA	Profit made by Chia	Profit made by Mbako	Profit made by Fru
1st	21,000	3,000	6,000	12,000
2nd	28,000	4,000	8,000	16,000
3rd	35,000	5,000	10,000	20,000

    Find;
(b) the total profit made in the 12th month,   **(2 marks)**
(c) the profit Mbako made in the 9th month,   **(2 marks)**
(d) the total profit Fru made in the first 12 months.   **(2 marks)**

---

4. (i) Given that a binary operation $*$ defined over the set $G = \{0, 1, 2, 3, 4\}$ by
    $x * y = (x + y) \, modulo \, 5$ forms a group
  (a) Copy and complete the table.   **(3 marks)**

*	0	1	2	3	4
0		1		3	4
1	1	2	3		0
2	2		4	0	
3	3	4	0	1	
4	4		1	2	3

    From the table,
(b) State the identity element,   **(1 mark)**
(c) State the inverse of each element.   **(1 mark)**

(ii) The transformation T is defined by the matrix M, where $M = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$.

  (a) Find the inverse of the matrix M.   **(3 marks)**
    Hence or otherwise,
  (b) find the point whose image is $(6, -5)$ **(2 marks)**

23/0575/2   Go on to the next page

5. Eposi has only 6,000 FCFA to buy her books and pens. She buys x books for 600 FCFA each and y pens for 300 FCFA.
   (a) Write down the total cost in terms of x and y of these materials and hence
      show that $2x + y \leq 20$. **(2 marks)**
      She intended to buy at least 4 books and decided to buy more than twice as many books as pens.
   (b) Write down further two inequalities involving x and y. **(2 marks)**
      On a graph paper, using the scale 2 cm to represent 2 units on the x- axis and 2 cm to represent
      4 units on the y- axis:
   (c) Shade, so as to leave unshaded , the region satisfied by these three inequalities and hence determine
      the minimum number of books and pens she buys. **(4 marks)**

6. (i) Solve, for x, in the range, $0° \leq x \leq 180°$, the equation $cosx - sin2x = 0$. **(3 marks)**

   (ii) A function f, of a real variable, θ, is defined by $f(\theta) = 2cos\theta + 3sin\theta$, for $0 \leq \theta \leq 2\pi$.

   (a) Copy and complete the table below. **(3 marks)**

θ	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(\theta)$		4		-2			2

   Taking $2cm$ to represent $\frac{\pi}{3}$ on the x-axis and $2cm$ to represent 1 unit on the y-axis,
   (b) Draw the graph of $y = f(\theta)$. **(2 marks)**
   (c) From your graph, state the maximum value of $y = f(\theta)$. **(1 mark)**

7. Given that the vector equations of the lines l_1 and l_2 are given by:
   $l_1: r = -i + 3j + s(3i - 2j)$,
   $l_2: r = 5i - j + t(2i + j)$, where s and t are constants respectively, intersect,

   (a) Find the value of $and\ t$. **(4 marks)**

   (b) Write down the position vector of the point of intersection of l_1 and l_2 **(1 mark)**

   (c) Find the cosine of the angle between l_1 and l_2. **(3 marks)**

8. (i) Given that $y = \frac{x^2}{1+x}, x \neq -1$, find $\frac{dy}{dx}$, simplifying your answer as far as possible. **(3 marks)**

   (ii) Evaluate $\int_0^{\frac{\pi}{2}} (2x - sin\ 2x)dx$ **(5 marks)**

SECTION B: MECHANICS

IF THIS SECTION IS CHOSEN, THEN SECTION C MAY NOT BE CHOSEN

(ANSWER ANY TWO QUESTIONS)

9. (i) Two particles A and B are moving on the coordinates axes OX and OY. At time t seconds, particle A
      is at the point with position vector $r = [(1 + 3t)i + 2t^2j]m$ and particle B is at point with
      position vector $r = [2t^2i - 3j]m$.
      Find,
      (a) the magnitude of the velocity of particle A relative to particle B when $t = 2$. **(4 marks)**

      (b) Show that the acceleration of B is constant **(2 marks)**

   (ii) Two particles, of masses $2kg$ and $3kg$ are moving towards each other with speeds $6ms^{-1}$
      and $2ms^{-1}$ respectively. After collision, they coalesce.
      Find;
      (a) their common velocity after collision **(3 marks)**

(b) the loss in kinetic energy due to the collision **(3 marks)**

(iii) Two particles M and N of masses 4 kg and 5kg are connected by a light inextensible sting passing over a smooth fixed pulley. The system is released from rest with the strings taut and hanging freely.
    (a) draw a diagram showing all the forces acting on the system **(1 mark)**
    (b) find the acceleration of the particles and the tension in the string. **(4 marks)**

---

10.   (i) The volume of a sphere is increasing at the rate of $25\pi cm^3 s^{-1}$. Given that the radius of the sphere is $5cm$, find the rate at which;
      (a) the radius is increasing, **(3 marks)**
      (b) the surface area is increasing. **(3 marks)**

$$\left[The\ volume\ of\ the\ sphere, V = \frac{4}{3}\pi r^3\ and\ the\ surface\ area\ of\ the\ sphere, A = 4\pi r^2\right]$$

(ii) The area bounded by the curve $y^2 = x^3$ and the x-axis and the lines $x = 2$ to $x = 4$ is rotated completely about the x-axis. Find the volume of the solid generated. **(5 marks)**

(iii) The position vectors of the centre of gravity of three particles of mass $mkg, 5kg$ and $3kg$ which are at the point with position vectors $3i - j, -6i + 2j$ and $-2i + 4j$ respectively is $-2i + aj$. Given that m and a are constants, find the values of m and a. **(6 marks)**

---

11.   (i)
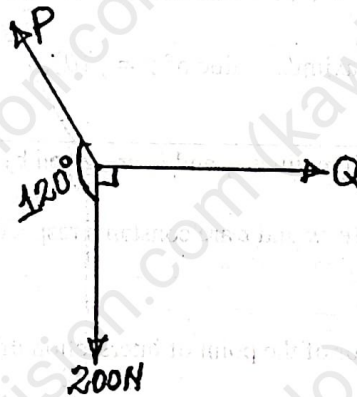
If the three forces $200N, P\ N\ and\ Q\ N$ as shown in the diagram above are in equilibrium, find the values of $P\ and\ Q$. **(6 marks)**

(ii)   A force $(3i - 2j)\ N$ displaces a particle, from a point A with position vector $5i + 2j$ to a point B with position vector $-3i + 8j$. Find the work done by the force in displacing the particle. **(5 marks)**

(iii)   A crane lifts a block of mass 5kg which is at rest from a horizontal ground vertically upward, to a height of 20m in 4 seconds. Find;
    (a)   the work done against gravity by the crane, **(3 marks)**
    (b)   the power generated by the crane. **(3 marks)**

SECTION C: STATISTICS AND PROBABILITY

(IF THIS SECTION IS CHOSEN, THEN SECTION B MAY NOT BE CHOSEN)

IF THIS SECTION IS CHOOSEN, THEN ANSWER ANY TWO QUESTIONS

12. The marks obtained by 60 students in an examination were grouped into classes and their corresponding frequencies listed in the distribution table below.

Marks (x)	0 - 9	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 -79
Nº of students (f)	5	7	8	11	13	8	6	2

    (a) Draw the cumulative frequency graph for the above distribution. **(5 marks)**

    (b) Hence, estimate the median mark **(3 marks)**
       Find,

    (c) The mean of the distribution, **(4 marks)**

    (d) Variance of the distribution. **(5 marks)**

---

13. (i) A discrete variable, X, has the following probability distribution

x	0	1	2	3	4
$P(X = x)$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	k	$\frac{1}{6}$

Find,

    (a) the value of k. **(3 marks)**

    (b) the mean and variance of the distribution. **(4 marks)**

    (c) $P(X \geq 2)$ **(3 marks)**

(ii) In a tray of eggs, 4 out of 8 are found to be bad.
    (a) Find the probability of selecting a bad egg at random from the tray. **(2 marks)**
      Given that 5 eggs are selected at random from the tray, using the binomial distribution, Find;
    (b) the probability that no bad egg was selected, **(2 marks)**
    (c) the mean and standard deviation of the binomial distribution. **(3 marks)**

---

14. (i) Given that $(A) = \frac{1}{10}, p(B) = \frac{2}{5}$ and that A and B are independent.
    Calculate:
    (a) $p(A \cap B)$ **(2 marks)**
    (b) $p(A \cup B)$ **(3 marks)**
    (c) $p(A' \cap B)$ **(3 marks)**

(ii) A bag contains 3 red and 3 white balls. Two balls are drawn from the bag, one after the other, without replacement.
    (a) Draw a tree diagram to illustrate all the possible outcomes. **(3 marks)**
      Using the tree diagram, calculate the probability that;
    (b) One ball of each colour is drawn, **(3 marks)**
    (c) At most one white ball is drawn. **(3 marks)**

GO BACK AND CHECK YOUR WORK