ADDITIONAL MATHEMATICS 2
0575

GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2024 **ORDINARY LEVEL**

Subject Title	**Additional Mathematics**
Paper No.	**2**
Subject Code No.	**0575**

Duration: Two and a Half Hours

Answer ALL QUESTIONS IN SECTION A and ANY TWO QUESTIONS FROM EITHER SECTION B or SECTION C.
IN SECTIONS B AND C, ALL QUESTIONS CARRY EQUAL MARKS.

Candidates are expected to answer a combination of Section A **and** Section B **OR** Section A **and** Section C but **NOT** a combination of all three.

All necessary working must be shown. No marks will be awarded for answers without brief statements showing how the answers have been obtained.

Electronic Calculators and Formulae Booklets are **ALLOWED.**

Where necessary take g as $10ms^{-2}$.

Turn Over

SECTION A: PURE MATHEMATICS
THIS SECTION IS COMPULSORY TO ALL CANDIDATES
(ANSWER ALL QUESTIONS)

1. (i) Find the value(s) of x for which for which $(\log_3 x)^2 - 4(\log_3 x) + 3 = 0$. **(4 marks)**

   (ii) Given that the roots of the quadratic equation $2x^2 - 3x - 5 = 0$ are α and β,
   a) find the values of $\alpha + \beta$ and $\alpha\beta$. **(1 mark)**
   b) write down another quadratic equation with integral coefficients whose roots are $-\alpha$ and $-\beta$. **(3 marks)**

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2. (i) Find the number of ways of arranging the letters of the word **ARRANGING**. **(4 marks)**

   (ii) Write down the first three terms in ascending powers of x in the binomial expansion of $(1 - 2x)^{-1}$, simplifying each term. **(4 marks)**

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3. During a meeting of women in a church, 24 women sat on the first row, 21 in the second row and 18 in the third row. Given that the sitting arrangement continues in the same pattern and no row was skipped, find,
   a) the number of women who sat on the 5th row. **(2 marks)**
   b) Show that no woman sat on the 9th row. **(3 marks)**
   c) Find the total number of women who attended the meeting. **(3 marks)**

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4 (i) The binary operation $*$ is defined over the set $G = \{1, 3, 5, 7\}$ where $*$ denotes multiplication modulo 8.
   a) Copy and complete the table below for $(G, *)$. **(3 marks)**

$*$	1	3	5	7
1	1	3	5	7
3	3			5
5	5	7		
7	7		3	

   b) State the identity element **(1 mark)**

(ii) A linear transformation T is given by
   $T: (x, y) \longmapsto (2x + y, x + 2y)$
   a) State the transformation matrix represented by T. **(1 mark)**
   b) Find the image of the point $P(1, 3)$ under T. **(2 marks)**
   c) Find the equation of the invariant line under T. **(2 marks)**

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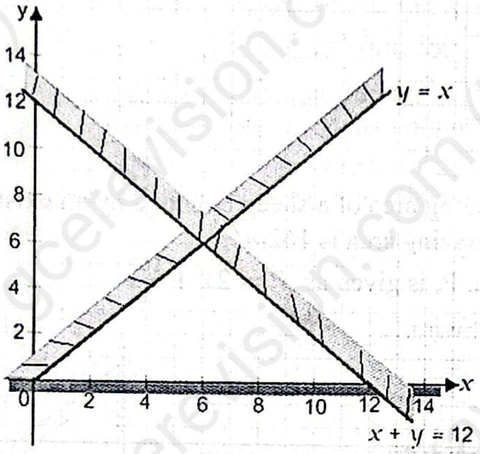
5. (i) A man sells x cups of patched groundnuts and y cups of patched corn during break in a certain school as shown in the inequality diagram below.



From the diagram,

a) write down three inequalities in terms of x and y that satisfy the unshaded region. **(2 marks)**

If he must sell both patched groundnuts and patched corn and that a cup of patched groundnut cost 125 francs and a cup of patched corn cost 50 francs,

b) find the maximum expenditure that can be incurred by the man. **(2 marks)**

(ii) The points A and B has coordinates $(-2, 5)$ and $(3, -1)$ respectively.

Find

a) the gradient of the straight line AB, **(2 marks)**

b) the equation of the straight line joining the points AB. **(2 marks)**

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6. (i) Solve for θ, in the range $0° \leq \theta \leq 360°$, the equation $\cos 2\theta = 1$ **(3 marks)**

(ii) The function $f(x)$ is defined by $f(x) = \sin x + \cos x$, where $0 \leq x \leq 2\pi$.

a) Copy and complete the table below. **(3 marks)**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$f(x)$	1		1	0		-1.4		0	

Taking 1cm to represent $\frac{\pi}{4}$ radians units on the x-axis and 2cm to represent 1 unit on the y-axis,

b) draw the graph of $y = f(x)$ **(2 marks)**

c) write down the minimum value of $f(x)$. **(1 mark)**

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7. The vector equations of the lines l_1 *and* l_2 are :

$r = i - j + \lambda(i - j)$

$r = 4i + (a + 1)j$ respectively, where λ and a are constants.

Find.

a) The value of a given that l_1 *and* l_2 intersect. **(3 marks)**

Hence find,

b) the position vector of their point of intersection **(2 marks)**

c) the angle between l_1 *and* l_2 **(3 marks)**

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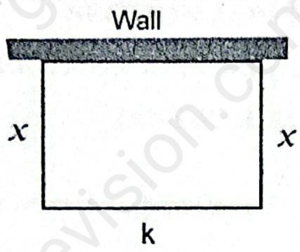
8. (i)



The diagram above shows a rectangular grazing area of a sheep bounded by an existing wall and barbwire on the sides labelled x and k. Given that the grazing area is $162m^2$.

a) Show that the perimeter of grazing area, P, is given as $P = 2x + \frac{162}{x}$ **(3 marks)**

b) Find the minimum perimeter of the barbwire. **(3 marks)**

(ii) $\int(3x - x^2)dx$. **(2 marks)**

SECTION B: MECHANICS

IF THIS SECTION IS CHOSEN, THEN SECTION C MAY NOT BE CHOSEN

(ANSWER ANY TWO QUESTIONS)

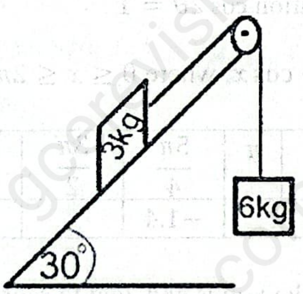
9. (i) The particle moves from the origin O along the x-axis, OX, so that its distance x cm from O at time, t seconds is given by $x = 2t^3 - 6t^2 + t$,

Find

a) The velocity of the particle after $t = 3$ seconds **(3 marks)**

b) The acceleration of the particle after $t = 3$ seconds **(3 marks)**

(ii)



Two particles P and Q of mass $6kg$ and $3kg$ respectively are connected by a light inextensible string which passes over a smooth fixed pulley at the top of a smooth plane inclined at 30° to the horizontal.

Given that the string is taut and the system is released from rest with particle 6kg hanging vertically, find,

a) their common acceleration. **(4 marks)**

b) the tension in the string. **(2 marks)**

(iii) A particle A, of mass $2kg$, moving with speed 3 ms^{-1} collides directly with a stationary particle B of mass $4kg$. Given that after collision A and B coalesce and move with a common speed v.

Find:

a) find the value of v. **(2 marks)**

b) the loss in kinetic energy due to the collision. **(3 marks)**

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10. (i) Air is escaping from a spherical balloon at the rate of $4cm^3s^{-1}$ at the instant when the radius is $2cm$. Find,

a) in terms of π, the rate of decrease of the radius, **(3 marks)**

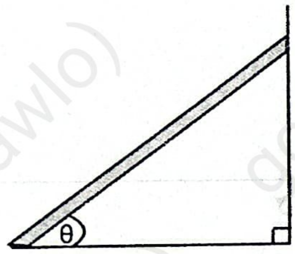
b) the rate of decrease of the surface area of the spherical balloon. **(2 marks)**

$\left[Volume\ of\ a\ spherical\ ballon\ is\ \frac{4}{3}\pi r^3\ and\ Surface\ area\ is\ 4\pi r^2\right]$

(ii) The area of the finite region between the curve $y^2 = 4 - x^2$, and the line $x = 0$ and $x = 2$ is rotated completely about the x-axis. Find the volume generated. **(6 marks)**

(iii) The position vectors of three particles of mass $2kg$, $3kg$ and $5kg$ are $(2\boldsymbol{i} + 3\boldsymbol{j})$, $(-3\boldsymbol{i} + 4\boldsymbol{j})$ and $(2\boldsymbol{i} - 5\boldsymbol{j})$ respectively. Find the position vector of the centre of gravity of these particles. **(6 marks)**

11. (i)



The figure above shows a uniform ladder of length $4m$ and weight w and it is placed with its lower end on a rough horizontal floor and the top end on a smooth vertical wall. Given that the ladder is inclined at an angle $\theta = 30^0$ to the horizontal floor and the coefficient of friction between the ladder and the floor is $\frac{1}{\sqrt{3}}$.

Find how far up the ladder a man of weight $2w$ can climb without the ladder slipping off. **(8 marks)**

(ii) A car of mass 1200kg has power of $6\,kW$. Given that the maximum speed of the car on a level road is $24ms^{-1}$

a) Calculate the total resistance to the motion of the car at this speed. **(4 marks)**

Given that the engine is still working at the same rate and that the resistance remains constant.

b) Calculate the acceleration of the car up a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{24}$, when the speed of the car is $6ms^{-1}$. **(5 marks)**

Turn Over

SECTION C: STATISTICS AND PROBABILITY

(IF THIS SECTION IS CHOSEN, THEN SECTION B MAY NOT BE CHOSEN)

IF THIS SECTION IS CHOOSEN, THEN ANSWER ANY TWO QUESTIONS

12. The quantity of maize, in kilogrammes (kg), harvested by 100 farmers are recoded as follows

Maize (kg)	5-9	10-14	15-19	20-24	25-29	30-34
Number of farmers (f)	2	29	37	16	14	2

(i) (a) Draw a cumulative frequency graph of the distribution **(5 marks)**

From your graph above, estimate

(b) the median **(3 marks)**

(c) the semi-interquartile range **(4 marks)**

(ii) Find the mean of the distribution **(5 marks)**

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13. (i) A discrete random variable, X, has probability mass function f defined by

$$f(x) = \begin{cases} k(4-x), & for\ x = 1, 2 \\ k(x-2), & for\ x = 3, 4, 5, \\ 0, & elsewhere \end{cases}$$

where k is a constant.

a) Copy and complete the distribution table below: **(2 marks)**

x	1	2	3	4	5
$P(X = x)$		$2k$			

Find

b) the value of the constant k **(2 marks)**

c) the mean and variance of X **(5 marks)**

(ii) A random variable, X is such that $X \sim Bin(n, \frac{1}{3})$ with mean 2.

Find:

a) The value of n **(2 marks)**

b) The standard deviation of the distribution **(3 marks)**

c) $P(X < 2)$ **(3 marks)**

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14. (i) Two events A and B are such that $P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(A \cap B) = \frac{2}{3}$.

Find

a) $P(A \cup B)$ **(3 marks)**

b) $P(B|A)$ **(3 marks)**

c) $P(A' \cap B')$ **(2 marks)**

(ii) A basket contains 20 white and 40 green cups. It is observed that 40% of the white cups and 30% of the green cups are breakable. A cup X is selected at random from the basket. Find the probability that

a) X is breakable. **(3 marks)**

b) X is a green cup or breakable. **(3 marks)**

c) X is a white cup given that it is breakable. **(3 marks)**

STOP

GO BACK AND CHECK YOUR WORK