1. (i) Find the set of values of x for which the roots of the quadratic equation $x^2 + (a-3)x + 2a - 1 = 0$ are real and distinct.

(ii) Given that $5^x = 2^y$ and that $2x + 3y = 6$, show that $x = \frac{\log 2}{\log 10}$ (7 marks)

2. (i) The first term of an arithmetic progression is $2a$ and the common difference is 3. If the sum of the first $2n$ terms is equal to the sum of the first n terms of this progression, express a in terms of n.

Obtain the value of a when $n = 5$ and hence find the sum of the first 10 terms of the progression.

(ii) Find the position of the term in x^{-18} in the expansion of $\left(x^4 + \frac{1}{x^2}\right)^{30}$ (10 marks)

3. (i) Differentiate with respect to x

(a) $\frac{(3x^2-2)}{(x^2+2)^2}$

(b) $\cos^2(3x-2)$

(ii) If $f(x) = 2x^4 - 4x^3 - 3x^2 + 1$, find $f'(x)$ and hence determine the set of values of x for which $f(x)$ is increasing (5, 6 marks)

4. (i) Find the general solution of the equation $cos^2 x = 2\sin x$.

(ii) $f(x) \equiv 2\cos x + 2\sin x$.

(a) Express $f(x)$ in the form $r\cos(x-\alpha)$ where $r > 0$ and $0° < \alpha < 90°$

(b) Find, to the nearest tenth of a degree, the set of values of x which satisfy the equation $2\cos x + \sin x = \sqrt{3}$

(c) Find the maximum value of $\frac{3}{7+\sqrt{5}f(x)}$. (4, 7 marks)

5. Vector parametric equations of the lines l_1 and l_2 are given by

$$l_1: r = 2i + j + \lambda(i + j + 2k), \qquad l_2: r = 2i + 2j + tk + \mu(i + 2j + k)$$

where t is a constant. Find

(a) The value of t for which l_1 and l_2 intersect. (4 marks)

(b) The position vector of the point of intersection of l_1 and l_2. (2 marks)

(c) The cosine of the acute angle between l_1 and l_2. (3 marks)

(d) The vector parametric equation of the plane containing l_1 and l_2. (2 marks)

6. (i) The function f is defined by $f(x) = \frac{2x-1}{x^2+2x-3}$,

(a) Express $f(x)$ in partial fractions.

Hence,

(b) Evaluate $\int_0^2 f(x)\,dx$ (6 marks)

(ii)) Sketch the curve of $y = x^3 - 3x^2 - 4x + 12$, showing clearly the intercepts with the coordinate axes and the behavior of the curve as it approaches its asymptotes. (5 marks)

7. (i) A function $f: \mathbb{R} \rightarrow \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers, is defined by $f(x) = ax^2 + bx + c$ for some constants a, b, and c.
Given that $f(1) = 3$ and $f(-1) = 7$,

(a) Find the values of the constants a, b and c

(b) Show that f is injective

(7 marks)

(ii) The table below shows corresponding values of x and y which approximately satisfy a relation of the form $y = ax^2 + b$, where a and b are constants

x	1	2	3	4
y	2.5	6.2	11.5	18.8

By drawing a suitable linear graph, determine the values of a and b, correct to one decimal place (7 marks)

8. (i) A basketball team of 8 players is to be selected from a group of 20 players in which there are three sisters. Find the number of ways in which the team can be selected if

(a) The three sisters must be included in the team,

(b) At most one of the three sisters is to be included in the team

(ii) The parametric equations of a curve are given by $x = 2t^2 - 3t + 1$, $y = t^3 - 2t^2 - t$, where t is a parameter.

Show that an equation of the tangent to the curve at the point with parameter $t = 2$ is $2x - 3y + 5 = 0$ (9 marks)

9. (i) The table below shows values of the continuous variable y corresponding to given values of x.

x	1	2	3	4
y	2.5	6.2	11.5	18.8

Use the trapezium rule to find an estimate for $\int_0^3 y\,dx$. (4 marks)

(ii) Show that the equation $x^3 + 2x^2 - 7x - 12 = 0$, has a root between 2 and 3.
Using the Newton-Raphson method and taking 2.5 as the first approximation, determine, by means of two iterations, two other approximations for the root, giving your answers correct to 3 decimal places. (5 marks)

10. Define the concept of equivalence relation.
A relation R is defined on the set $\mathbb{Z}$, of integers, by ab is even.
Show that R is an equivalence relation. Write down 3 elements of the relation R where (8 marks)
aRb but $a \neq b$.