BAMENDA ARCHDIOCESAN EXAMINATION BOARD (BAEBOC)
MOCK G.C.E. EXAMINATION

MARCH 2025 **ADVANCED LEVEL**

Subject Title	**FURTHER MATHEMATICS**
Paper Number	**PAPER 2**
Subject Code	**0775**

Time: THREE HOURS

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the Board and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

TURN OVER

1. a) Given the differential equation $\frac{d^2y}{dx^2} + 4y = 8x + 4$,
   i) show that the particular solution is $p(x) = 2x + 1$. (2mks)
   ii) find the solution of the differential equation for which $\frac{dy}{dx} = 1$ and $y = 1$ when $x = 0$. (3mks)

   b) Find the cartesian equation of the plane containing the lines $l_1: r = (9i - 4j + 5k) + \mu(4i - j + 2k)$ and $l_2: r = (4i - 3j + 2k) + \alpha(-3i + j - k)$. (4mks)

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2. a) Given the group (G, *) and $x * y = e$, where $e, x, y \in G$ and e is the identity element, prove that $x = y^{-1}$ and $y = x^{-1}$. (3mks)

   b) Construct the operation table for $(G, \times_{14})$, where $G = \{2,4,6,8,10,12\}$ and $\times_{14}$ is multiplication modulo 14. (2mks)

   Hence, or otherwise, solve
   i) completely the equation $x * y = e$, where e is the identity element and $x \neq y$.
   ii) $x^2 = e$, (3mks)

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3. a) A curve C has equation $y = \frac{x^3}{6} + \frac{1}{2x}$ for $1 \leq x \leq 3$. Show that the length of the curve C is $\frac{14}{3}$ units. (4mks)

   b) Given that $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, prove by induction that $A^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ n & n & 1 \end{pmatrix}$, $n \in \mathbb{Z}^+$. (4mks)

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4. a) Express $f(x) = \frac{1+x^2+x^3}{x^2(1+x^2)}, x \neq o$, in partial fractions.

   Hence, show that $\int_1^3 f(x)dx = \frac{1}{2}ln5 + \frac{2}{3}$ (6mks)

   b) Given that $I_n = \int_{-1}^{0} x(1+x)^n dx$, show that $(n+2)I_n = nI_{n-1}$. (4mks)

   Evaluate I_3. (2mks)

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5. The curve D has equation $y^2 - 4x^2 + 4y + 8x - 4 = 0$.
   i) Express the curve D in the form $\frac{(y+y_1)^2}{a^2} + \frac{(x+x_1)^2}{b^2} = 1$, stating the values of a^2, b^2, y_1 and x_1. (3mks)
   ii) Show that the asymptotes to the curve D are $y = 2(x-2)$ and $y = -2x$. (2mks)
   iii) Show ,also, that the point P(1,0) lies on D and find the equation of the tangent to the curve D at the point P. (3mks)

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6. a) Use the Euclidian Algorithm to find the g.c.d., h, of 2695 and 1547.Express the gcd in the form $h = 2695p + 1547q$, where $p, q \in \mathbb{Z}$

   Hence, or otherwise, solve the linear congruence equation $54x \equiv 7(mod\ 31)$ (6mks)

   b) Solve the equation $2coshx - sinhx = 2$. (3mks)

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7. The equations of two polar curves C_1 and C_2 are C_1: $r = \frac{6}{1+cos\theta}, -\pi \leq \theta \leq \pi$,

   $C_2: r = 2sec\theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

   i) Find the polar coordinates of the point(s) of intersection of C_1 and C_2. (2mks)
   ii) Find the Cartesian equations of C_1 and C_2 and, on the same axes, sketch C_1 and C_2. (4mks)
   iii) Find the Cartesian equation of the normal to C_1 at the point $P(2, 2\sqrt{3})$. (3mks)

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8. A function, f is defined by $f(x) = x^2e^{-x^2}$.

Find i) the domain of f (1mk)

ii) $\lim_{x\to-\infty}(fx)$ and $\lim_{x\to+\infty}(fx)$ (2mks)

iii) the asymptote to the curve $f(x)$ (1mk)

iv) the turning point(s) of f(x). (3mks)

Hence, v) draw a variation table for f, (2mks)

vi) state the bounds of f. (1mk)

vii) sketch the graph of $y = f(x)$ (2mks)

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9. The sequences (U_n), (V_n) and (W_n) are defined as follows:

$\begin{cases} U_0 = 1 \\ U_{n+1} = 3V_n \end{cases}$, $\begin{cases} V_0 = 2 \\ V_{n+1} = -U_n + 4V_n \end{cases}$ and $W_n = U_n - U_{n-1}$

i) Find U_1 (1mk)

Show that ii) $U_{n+1} = 4U_n - 3U_{n-1}$ (2mks)

iii) $U_n = \frac{3}{2}[5(3^{n-1}) - 1]$ (3mks)

iv) (W_n) is a geometric sequence and express W_n in terms of n, (3mks)

Another sequence (T_n) is defined by $T_n = V_n - U_n$. Show that

v) (T_n) is a constant sequence. (2mks)

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10. a) A similarity transformation (similitude) is defined by $z' = 2z + 3 - i$.

i) Find the invariant point under this transformation. (2mks)

ii) Given the object triangle ABC in the z-plane where A(0,0),B(2,2) and C(2,0),on the same axes, draw triangle ABC and its image $A'B'C'$ under this transformation. (4mks)

b) Given that $z = \cos\theta + i\sin\theta$, show that

i) $z + \frac{1}{z} = 2\cos\theta$ (1mk)

ii) $z^n + \frac{1}{z^n} = 2\cos n\theta$ (1mk)

Hence , or otherwise show that

iii) $16\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta$. (3mks)

iv) $\int_0^{\frac{\pi}{6}} \cos^5\theta \, d\theta = \frac{203}{480}$ (3mks)

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