0775/2/2025
F. MATHS A/L

SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION

THE TEACHERS' RESOURCE UNIT (TRU)

IN COLLABORATION WITH

THE REGIONAL INSPECTORATES OF PEDAGOGY FOR SCIENCE

AND

THE SOUTH WEST ASSOCIATION OF MATHEMATICS TEACHERS (SWAMT)

Wednesday 26/03/2025: Afternoon ADVANCED LEVEL

Subject Title	**FURHER MATHEMATICS**
Subject Code Number	**0775**
Paper Number	**Paper 2**

THREE HOURS

INSTRUCTIONS TO CANDIDATES

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the GCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. A particular integral to the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=2\sin 4x$ is $C\sin 4x+D\cos 4x$.

(a) Find the values of the constants C and D. (4 marks)

(b) Given that when $x=0$, $y=0$ and $\frac{dy}{dx}=1$, find the particular solution to the differential equation.

(5 marks)

2. (i) Show that the set $C=\{1,i,-1,-i\}$, together with the operation $\times$ (multiplication), forms a group, where $i^2=-1$.

Hence, show that the group is a cyclic group. (7 marks)

(ii) Find the remainder when 5^{119} is divided by 59. (4 marks)

3. (i) Express $f(x)=\frac{4x-1}{(x^2+1)x^2}$, $x\neq 0$, in partial fractions. (3 marks)

(ii) The line L is defined by $L:\vec{r}=9\vec{i}-\vec{j}+8\vec{k}+\mu(3\vec{i}-\vec{j}+2\vec{k})$, $\mu\in\mathbb{R}$.

Find

(a) the coordinates of the foot of the perpendicular from the point $A(1,2,4)$ to L. (2 marks)

(b) the perpendicular distance from the point A to L. (2 marks)

(c) the position vector of the reflection of A in L. (2 marks)

4. A curve is given parametrically by the equations $x=-t+\cosh t$, $y=t+\cosh t$.

Show that its arc length for $0\le t\le\frac{1}{2}\ln 2$ is $\frac{1}{2}$. (6 marks)

5. (i) A transformation f on a complex plane is defined by $Z'=4iZ+5$.

(a) Show that f is a similitude and find its radius. (2 marks)

(b) Find the inverse transformation f^{-1}. (2 marks)

(ii) (a) Find the Cartesian equation of the curve whose polar equation is $r=k\cos\theta$, $k\in\mathbb{R}$. (3 marks)

(b) Identify the curve. (1 mark)

6. (i) The hyperbola H has equation $\frac{x^2}{16}-\frac{y^2}{4}=1$.

The line l_1 is the tangent to H at the point $P(4\sec t, 2\tan t)$.

(a) Show that an equation of l_1 is $2y\sin t = x - 4\cos t$. (3 marks)

The line l_2 passes through the origin and is perpendicular to l_1.

The line l_1 and l_2 intersect at the point Q.

(b) Show that, as t varies, an equation of the locus of Q is $(x^2+y^2)^2 = 16x^2 - 4y^2$. (3 marks)

(ii) Given that $f(x)=\frac{1}{\sqrt{4x^2+9}}$, show that $\int f(x)dx = A\sinh^{-1}(Bx)+C$, where C is an arbitrary constant and A and B are constants to be determined. (Use the substitution $2x = 3\sinh u$.) (3 marks)

7. (i) Given that $I_n = \int_0^{\frac{\Pi}{4}} \tan^n x dx$, show that, for $n \geq 2$, $I_n + I_{n-2} = \frac{1}{n-1}$. (4 marks)

(ii) A sequence of numbers is defined by $U_1 = 6$, $U_2 = 27$, $U_{n+2} = 6U_{n+1} - 9U_n$, $n \geq 1$.

Prove by mathematical induction that, for $n \in Z^+$, $U_n = 3^n(n+1)$. (5 marks)

8. A function, f, is defined by $f(x) = \ln\left(\frac{2x-8}{x-2}\right)$.

(a) Find the domain of f. (2 marks)

(b) Evaluate $\lim_{x\to-\infty} f(x)$, $\lim_{x\to+\infty} f(x)$, $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 4^+} f(x)$.

Hence, state the equations of the 3 asymptotes to the curve $y = f(x)$. (4 marks)

(c) Find $f'(x)$ and determine whether or not the curve $y = f(x)$ has a turning point. (3 marks)

(d) Draw the variation table for f. (1 mark)

(e) Determine the x and y intercepts of the curve $y = f(x)$. (1 mark)

(f) Sketch the curve $y = f(x)$. (2 marks)

(g) State the centre of symmetry of the curve $y = f(x)$. (1 mark)

9. A sequence, (U_n), is defined recursively by $U_0 = e^3$, $U_{n+1} = e\sqrt{U_n}$, $n \in \mathbb{N}$.

(a) Calculate U_1 and U_2. (3 marks)

Suppose another sequence, (V_n), is defined by $V_n = \ln U_n - 2$,

(b) show that (V_n) is a geometric sequence, stating its first term and its common ratio. (4 marks)

(c) Deduce V_n and hence, U_n in terms of n. (3 marks)

(d) Calculate the limit of (V_n) and (U_n). (4 marks)

10. (i) A transformation from the $z-$ plane to the $w-$ plane is given by $w = z^2$.

(a) Show that the line with equation $\text{Im}(z) = 1$ in the $z-$ plane is mapped to a parabola in the $w-$ plane, giving an equation for this parabola. (4 marks)

(b) Sketch the parabola on an argand diagram. (2 marks)

(ii)(a) Given that $|z| < 1$, write down the sum of the infinite series $1 + z + z^2 + z^3 + \ldots$ (1 mark)

Given also that $z = \frac{1}{2}(\cos\theta + i\sin\theta)$,

(b) use De Moivre's theorem, to prove that $\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta + \frac{1}{8}\sin 3\theta + \ldots = \frac{2\sin\theta}{5 - 4\cos\theta}$. (3 marks)

(c) show that the sum of the infinite series $1 + z + z^2 + z^3 + \ldots$ cannot be purely imaginary. (2 marks)

END