0770/2/2025
P.M.S. A/L

SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION

THE TEACHERS' RESOURCE UNIT (TRU)

IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR SCIENCE

AND

THE SOUTH WEST ASSOCIATION OF MATHEMATICS TEACHERS (SWAMT)

Friday 28/03/2025: Morning

ADVANCED LEVEL

Subject Title	**PURE MATHEMATICS WITH STATISTICS**
Subject Code Number	**0770**
Paper Number	**Paper 2**

THREE HOURS

INSTRUCTIONS TO CANDIDATES

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

You are reminded for the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the GCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. (i) Given that $\alpha^2\beta$ and $\beta^2\alpha$ are the roots of the quadratic equation $x^2 - 2x + 27 = 0$
Find the quadratic equation whose roots are α and β. **(4 marks)**

(ii) Given the polynomial $f(x) = 2x^3 - x^2 + ax + b,\ a, b \in \mathbb{R}$. The remainder when $f(x)$ is divided by $(x + 1)$ is three times the remainder when divided by $(x - 1)$ and that $f(0) = 1$.
Find the values of a and b. **(5 marks)**

(iii) Given that $\log_3 6 = x$ and $\log_6 5 = y$, show that $\log_3 10 = x(y + 1) - 1$ **(3 marks)**

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2. (i) The function f is defined by

$$f: \mathbb{R} - \{-1\} \to \mathbb{R} - \{2\}, \quad x \mapsto \frac{2(x-1)}{x+1}$$

(a) Show that f is bijective. **(4 marks)**

(b) Find the function g such that $f \circ g(x) = \frac{1}{x},\ x \neq 0$. **(3 marks)**

(ii) Find the constant term in the expansion of $\left(5x - \frac{1}{2x}\right)^8$ **(4 marks)**

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3. (i) Given the vectors

$$\mathbf{a} = (2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}), \quad \mathbf{b} = (-\mathbf{i} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{c} = (2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Find

(a) $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$ and hence, state a vector perpendicular to $\mathbf{a}$. **(3 marks)**

(b) the Cartesian equation of the plane π containing $\mathbf{a}$ and $\mathbf{b}$. **(2 marks)**

(c) the perpendicular distance from $\mathbf{c}$ to the plane π. **(2 marks)**

(ii) Write down the power set of the set $A = \{1, 5, 7\}$. **(2 marks)**

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4. (i) The angle θ is such that

$$\sin\left(2\theta + \frac{\pi}{4}\right) = 3\cos\left(2\theta + \frac{\pi}{4}\right)$$

(a) Show that $\tan 2\theta = \frac{1}{2}$ **(2 marks)**

(b) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is obtuse. **(2 marks)**

(ii) Express $f(x) = 7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and α is an acute angle. **(3 marks)**

Hence, state the range of the function $g(x) = 12 + 7\cos x - 24\sin x,\ 0 \leq x \leq 2\pi$. **(2 marks)**

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5. (i) Find $\frac{dy}{dx}$ given that

(a) $y = \ln\left[\frac{(x+1)^3}{x}\right]$ **(3 marks)**

(b) $x = 2t,\ y = \frac{2}{t}$ where t is a paramter. **(3 marks)**

(ii) Given that the function $f(x) = x^3 - 1$ is continuous and differentiable in the interval $(-2,\ 2)$.
Find the x-coordinates of the points where the tangent to the curve $y = f(x)$ is parallel to the chord joining the points $(-2,\ -9)$ and $(2,\ 7)$. **(5 marks)**

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6. (i) (a) State De Moivre's theorem **(1 mark)**

(b) Using De Moivre's theorem, or otherwise, express the complex number $z = \left(1 + i\sqrt{3}\right)^{10}$ in the form $a + ib$, where $a, b \in \mathbb{R}$. **(6 marks)**

(ii) Show that the y-axis is a tangent to the circle $C: \; x^2 + y^2 - 2x + 4y + 4 = 0$. **(4 marks)**

7. (i) Using the substitution $x = 2\cos\theta$, show that

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4 - x^2}}\,dx = \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2\theta\, d\theta$$ **(4 marks)**

Hence, deduce that $\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4 - x^2}}\,dx = \frac{1}{4}(\sqrt{3} - 1)$ **(2 marks)**

(ii) Solve the differential equation

$$\frac{dy}{dx} = e^{x+2y}$$

given that $y = -\frac{1}{2}\ln 2$ when $x = 0$, leaving your answer in the form $y = f(x)$. **(5 marks)**

8. (i) The first term of an arithmetic progression is 2. The ratio of the second to the seventh terms is $1:4$. Find the common difference of the progression. **(3 marks)**

(ii) Given the matrices $\mathbf{A} = \begin{pmatrix} 2 & 1 & -2 \\ 2 & 3 & 1 \\ -1 & 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 & -2 & 7 \\ -5 & 2 & -6 \\ 3 & -1 & 4 \end{pmatrix}$

Find the matrix products **AB** and **BA**. **(3 marks)**

Hence,

(a) state $\mathbf{A}^{-1}$, the inverse of **A**. **(1 mark)**

(b) find the point whose image is $(1, 0, 2)$ under the transformation represented by the matrix **A**. **(2 marks)**

9. The function f is defined by

$$f(x) = \frac{ax + 1}{x - b}, \quad a, b \in \mathbb{R}, \qquad x \neq b$$

Given that the lines $x = 1$ and $y = 2$ are the asymptotes of $f(x)$.

(a) Find the values of a and b. **(3 marks)**

Hence,

(b) Show that the graph of $y = f(x)$ has no turning points. **(2 marks)**

(c) Study the monotonicity of f in its domain. **(2 marks)**

(d) Sketch the graph of $y = f(x)$, showing clearly its intercepts with the coordinate axes and behavior near its asymptotes. **(2 marks)**

10. (i) Given the statements

p : x is a prime number
q : x is less than 2

Write the following in ordinary English;

(a) $p \wedge \sim q$ **(1 mark)**

(b) $\sim (q \Rightarrow p)$ **(1 mark)**

(c) the contrapositive of $p \Rightarrow \sim q$ **(1 mark)**

(ii) Given the function $f(x) = 2x^3 + x^2 - 1$.

(a) Show that $f(x) = 0$ has a solution in the interval $(0, 2)$. **(3 marks)**

(b) If x_0 is a first approximation to the roots of the equation $f(x) = 0$, use the Newton-Raphson's method to show that a second approximation is given by

$$x_1 = \frac{4x_0^3 + x_0^2 + 1}{6x_0^2 + 2x_0}$$

Hence, using $x = 1$ as a first approximation to the roots of the equation, obtain a second approximation to the roots. **(3 marks)**

END