GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2025 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Statistics**
Paper No.	**Paper 2**
Subject Code No.	**0770**

Duration: Three Hours.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

Turn Over

1. (i) When the polynomial $P(x)$, where $P(x) = 2x^3 + ax^2 + bx - 12$ is divided by $(x + 3)$ the remainder is 18. Given that $(x + 1)$ is a factor of $P(x)$,
   (a) find the values of the constants a and b.
   (b) factorise $P(x)$ completely. **(9 marks)**

   (ii) Find the range of values of $k \in \mathbb{R}$, for which the roots of the quadratic equation $x^2 + kx + 3 + k = 0$ are real and distinct. **(3 marks)**

---

2. The table below shows some particular values of two variables x and y.

x	2	13	24	37	52
y	3.1	5.4	6.5	7.6	8.2

It is known that x and y satisfy a relation of the form $y = ax^n$. By drawing a suitable linear graph relating $\log y$ and $\log x$, find the values of the constants of a and n, giving the answer to one decimal place. **(8 marks)**

---

3. (i) Show that $\frac{\sin 3\theta + \sin \theta}{\cos 3\theta + \cos \theta} \equiv \tan 2\theta$ **(3 marks)**

   (ii) Given that $f(x) = \cos x + \sqrt{3} \sin x$,
   (a) Express $f(x)$ in the form $R\cos(x - \lambda)$, where $R > 0$ and $0 < \lambda < \frac{\pi}{2}$.
   Hence, find
   (b) the minimum value of $\frac{1}{1 + |f(x)|}$,
   (c) the general solution of $f(x) = \sqrt{3}$. **(8 marks)**

---

4. (i) The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms of the progression is 57.
   Find the number of terms in the progression. **(4 marks)**

   (ii) Find the numerical value of the term independent of x in the expansion of $\left(3x - \frac{1}{x^2}\right)^9$. **(4 marks)**

---

5. (i) Express the complex number $z = \frac{-5 + 10i}{1 + 2i}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.
   Hence or otherwise, find the modulus and the argument of z^3. **(6 marks)**

   (ii) Given that $f(x) = x^3 - 2x - 11$, show that the equation $f(x) = 0$ has a root between 2 and 3. Taking 2 as a first approximation to the root of the equation $f(x) = 0$, use one iteration of the Newton-Raphson procedure to obtain a second approximate root, giving the answer to one decimal place. **(6 marks)**

---

6. (i) The function f is periodic with period 4. Given that $f(x) = \begin{cases} x^2 - 2 \text{ for } 0 \le x < 2, \\ 4 - x \text{ for } 2 \le x \le 4, \end{cases}$
   (a) evaluate $f(19)$ and $f(-43)$
   (b) sketch the graph of $f(x)$ in the range $-4 \le x < 8$. **(6 marks)**

   (ii) A relation $\mathcal{R}$ is defined on $\mathbb{Z}$, the set of integers, by $a\mathcal{R}b$ if and only if $(a + b)$ is even. Show that $\mathcal{R}$ is an equivalence relation. **(6 marks)**

7. (i) Two statements p and q are defined as:
p: Kelvin will go to Kribi.
q: Kelvin will visit the seaport.
Translate into ordinary English, the statements:
(a) $p \Rightarrow q$
(b) $p \wedge q$
(c) $\sim p \vee \sim q$
(3 marks)

(ii) The position vectors of the points A , B and C are $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$, $\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ and $7\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ respectively.
Find
(a) the vector equation of the line AB.
(b) the cartesian equation of the plane ABC.
(c) the sine of the angle between line AB and plane ABC.
(8 marks)

---

8. (i) Given that $3x^2 - 4xy + 2y^2 - 6 = 0$, show that $\frac{dy}{dx} = \frac{3x - 2y}{2x - 2y}$. **(3 marks)**

(ii) Given that $f(x) = \frac{x + 10}{x^2 - x - 12}$, express $f(x)$ in partial fractions.

Hence, show that $\int_5^7 f(x)\,dx = \ln\left(\frac{36}{5}\right)$. **(7 marks)**

---

9. (i) A matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$.
Find
(a) $|\mathbf{M}|$, the determinant of matrix **M**.
(b) $\mathbf{M}^{-1}$, the inverse of matrix **M**. **(5 marks)**

(ii) Prove by mathematical induction that for all $n \in \mathbb{N}$, $\sum_{r=1}^{n} r(3r - 1) = n^2(n + 1)$. **(5 marks)**

---

10. (a) Solve the differential equation $(x - 1)\frac{dy}{dx} = 1 - y$, given that $y = -2$ when $x = 2$, expressing the result in the form $y = f(x)$. **(5 marks)**

(b) Sketch the curve $y = \frac{x - 4}{x - 1}$, showing clearly the points where the curve crosses the coordinate axes and the behaviour of the curve near its asymptotes. **(4 marks)**
