ETA SCHOOLS FOR BRIGHT AND GIFTED CHILDREN KUMBA

Eta College Mock Examination, 2025/2026

MARCH 2026 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**2**
Subject Code No.	**0775**

Three Hours

Answer ALL 10 questions

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the board, and noiseless non programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. Using the substitution $v = \dfrac{dy}{dx}$, where v is a function in x, show that the differential equation

$$\left(x^2+1\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \frac{6}{1+x^2} = 0$$

can be transformed to the differential equation $\dfrac{dv}{dx} + \dfrac{2xv}{x^2+1} = -\dfrac{6}{\left(x^2+1\right)^2}$. **(3 marks)**

Hence, or otherwise, show that if $y = 2$, $\dfrac{dy}{dx} = 0$ when $x = 0$, then $y = 2 - 3\left(\tan^{-1}x\right)^2$. **(5 marks)**

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2. Given that $f(x) = \dfrac{x^4+2x^2+x+3}{(x+2)\left(x^2+1\right)^2}$,

(a) Show that $\left(x^2+1\right)^2 + x + 2 \equiv x^4 + 2x^2 + x + 3$ **(1 mark)**

Hence, or otherwise,

(b) express $f(x)$ in partial fractions. **(2 marks)**

(c) show that $\int_0^1 f(x)dx = \ln\left(\dfrac{3}{2}\right) + \dfrac{1}{8}(2+\pi)$. **(5 marks)**

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3. (i) Prove by mathematical induction, or otherwise, that $7^n(2n+1)+11$ is divisible by 4, for all $n \in \mathbb{N}$, $n \geq 1$ **(5 marks)**

(ii) The set $G = \{1,\ 5,\ 8,\ 12\}$ forms a group under multiplication modulo 13.

(a) Draw up the operation table for the group. **(2 marks)**

(b) Find $5^4(\text{mod}13)$, hence state a generator of the group. **(3 marks)**

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4. (i) Using the Intermediate Value Theorem, show that the equation $e^x + 4x - 2 = 0$ has a unique solution in the interval $[0,\ 1]$ **(4 mark)**

(ii) Prove that $\coth^{-1}(2x) = \dfrac{1}{2}\ln\left(\dfrac{2x-1}{2x+1}\right)$ **(3 marks)**

Hence, investigate the parity of $f(x) = \coth^{-1}\left(\dfrac{2x}{3}\right)$, and evaluate $\int_{-1}^{1}\coth^{-1}\left(\dfrac{2x}{3}\right)dx$ **(3 marks)**

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5. (i) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^n$ is defined by the matrix $\mathbf{M} = \begin{pmatrix} 3 & 1 & -1 \\ 6 & 2 & -2 \\ 3 & 1 & -1 \end{pmatrix}$

(a) Show that the transformation maps $\mathbb{R}^3$ onto a line and find the equation of the line. **(5 marks)**

(b) Find the kernel of T in the form $\{\lambda\mathbf{a} + \mu\mathbf{b}\}$, $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$. **(3 marks)**

(ii) Prove that the vectors $\mathbf{a} = -\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ are linearly independent. **(2 marks)**

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6. The lines L_1 and L_2 are defined by

$$L_1 : \mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(-6\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$$

$$L_2 : \mathbf{r} = 4\mathbf{j} + 5\mathbf{k} + t(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

(a) Show that L_1 and L_2 are parallel. **(2 marks)**

(b) Find the Cartesian equation of the plane containing L_1 and L_2. **(3 marks)**

(c) Find the coordinates of the foot of the perpendicular from the point $P(2,\ 4,\ 1)$ to the line L_2 **(3 marks)**

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7. The numerical sequence u_n is defined in $\mathbb{N}$ by $u_0 = 2$, $u_{n+1} = \dfrac{3+u_n}{5-u_n}$

(a) Show that $u_{n+1} - 3 = \dfrac{4(u_n - 3)}{2 + (3 - u_n)}$ **(2 marks)**

(b) Prove by mathematical induction that $u_n < 3$, $\forall n \in \mathbb{N}$. **(3 marks)**

Another sequence v_n, for $n \in \mathbb{N}$, is defined by $v_n = \dfrac{u_n - 1}{3 - u_n}$

(c) Show that the sequence v_n is geometric with common ratio $\dfrac{1}{2}$. **(3 marks)**

(d) Express v_n in terms of n and show that $\lim\limits_{n\to\infty}(u_n) = 1$ **(2 marks)**

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8. (i) Using the Chinese remainder theorem, or otherwise, solve the congruences

$$3x \equiv 1 \pmod 7$$
$$3x \equiv 2 \pmod 8$$
$$4x \equiv 3 \pmod{11}$$

(5 marks)

(ii) The Cartesian equation of a hyperbola is given by $\dfrac{x^2}{9} - \dfrac{y^2}{3} = 1$

Find

(a) the eccentricity, **(2 marks)**

(b) the polar equation of its asymptotes **(2 marks)**

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9. (i) Solve the complex equation $z^4 = 16i$ **(3 marks)**

(ii) A similitude, s_1, transforms every point $M(x,\ y)$ to $M'(x',\ y')$, where

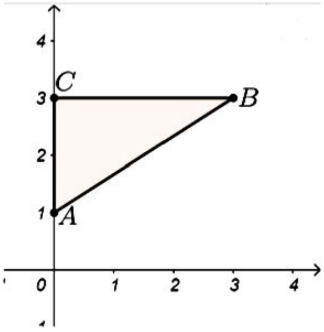
$$x' = x - y - 2$$
$$y' = x + y + 1$$

(a) Find the image of the line $y = 2x$ under the transformation s_1, **(2 marks)**

(b) Hence, deduce the angle of rotation of the similitude, **(2 marks)**

(c) Show that the complex representation of s_1 is given by $z' = (1+i)z - 2 + i$, and state the scale factor and centre. **(3 marks)**

On the same axes, copy triangle ABC below and its image $A'B'C'$ under s_1 **(3 marks)**



Another similitude, s_2 is defined by $z' = iz - 2 - 3i$

(d) Find $s_1 \circ s_2$ in the form $z' = az + b$, $a, b \in \mathbb{C}$. **(3 marks)**

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10. The function g is defined on $]0, +\infty[$ by $g(x) = x^2 + 2 - 2\ln x$

(a) Find $g'(x)$ and study the sign of $g(x)$ **(2 marks)**

(b) Hence, deduce that $g(x) > 0$, $\forall x \in]0, +\infty[$ **(2 marks)**

Another function f is defined on $]0, +\infty[$ by $f(x) = x - 1 + \frac{2\ln x}{x}$

(c) Evaluate $\lim\limits_{x \to 0^+} f(x)$, $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to +\infty} [f(x) - (x-1)]$

Hence, write down the equations of asymptotes of f. **(4 marks)**

(d) Investigate the relative position of the graph of $y = f(x)$ and the line $y = x - 1$. **(3 marks)**

(e) Show that $f'(x) = \frac{g(x)}{x^2}$ and find $f''(x)$ **(2 marks)**

(f) Find the coordinates of the point of inflexion of $f(x)$ **(2 marks)**

(g) Find the intervals in which f is concave up and where it is concave down. **(3 marks)**

(h) Draw the variation table of f and sketch the graph of $y = f(x)$. **(2 marks)**

END