

765 PURE MATHEMATICS WITH MECHANICS

770 PURE MATHEMATICS WITH STATISTICS

INTRODUCTION

The syllabus has been constructed so that it assumes knowledge of the topics contained in syllabus for the subject Mathematics 570 at Ordinary level. It has been done to emphasise the importance of the core of Pure Mathematics and its application in either mechanics for the 765 syllabus or statistics for the 770 syllabus

AIMS

The aims of the two Advanced Level syllabuses for the subjects, 765 Pure Mathematics with Mechanics and 770 Pure Mathematics with Statistics are:

- i) to provide a broad conspectus of mathematics to those who may not study mathematics beyond this level for whose course of study requires a knowledge of advanced level mathematics.
- ii) to enable students to acquire knowledge and skills with confidence, satisfaction and enjoyment.
- iii) to give students experience of mathematical activity and investigation, and develop resourcefulness in solving problems for which a ready method is not available.
- iv) to enable students to apply mathematics and recognize its significance to other disciplines.
- v) to develop students' understanding of mathematical reasoning and processes and the ability to relate different areas of mathematics to one another.
- vi) to analyse problems logically, recognise when and how a situation may be represented mathematically, identify relevant factors where necessary and select an appropriate mathematical method to solve the problem.
- vii) to use mathematics as a means of communication with emphasis on the use of clear expression.
- viii) to have mathematical background necessary for further studies in this or related subjects.

Restriction on Subject Combinations

765 Pure Mathematics with Mechanics (PMM) **should not** be taken by candidates taking 770 Pure Mathematics with Statistics (PMS) at the Advanced Level.

GENERAL OBJECTIVES

The objectives of the examinations are to test the ability of all candidates to:

- i) demonstrate a confident knowledge of the techniques of Pure Mathematics specified in the syllabus.
- ii) demonstrate an understanding of the concepts and principles of mechanics and

- probability.
- iii) apply their knowledge of mathematics to solve problems for which an immediate solution is not available and which involve a knowledge of more than one topic of the syllabus.
 - iv) write clear accurate mathematics solutions.
 - v) demonstrate an understanding of the nature of proofs and disproof in mathematics.

STRUCTURE OF THE EXAMINATION

Weighting of the Assessment objectives

Assessment Objectives	Weighting of Assessment Objectives
1. Knowledge and understanding) (AO1).	10 %
2. Application of knowledge (AO2).	20 %
3. Analysis (AO3).	30 %
4. Synthesis (AO4).	30 %
5. Evaluation (AO5).	10 %

Scheme of Assessment

Paper Number	Mode of Assessment	Number of Questions	Number of questions to be answered	Duration	Weighting
1	50 MCQ	50	50	1½ hours	20 %
2	Essay questions	10	10	2½ hours	40 %
3	Essay questions	8	8	2½ hours	40

Table of Specification (TOS)

Paper Number	Category	Number of questions	Marks	Level of difficulty
1	Knowledge	5	5	*
	Comprehension			*
	Application	10	10	*
	Analysis	15	15	**

Paper Number	Category	Number of questions	Marks	Level of difficulty
	Synthesis	15	15	**
	Evaluation	5	5	***
2	Knowledge	1	10	*
	Comprehension			
	Application	4	40	**
2 cont.	Analysis	4	40	**
	Synthesis	1	10	***
	Evaluation			
3	Knowledge	1	10	*
	Comprehension	3	40	**
	Application			
	Analysis			
	Synthesis	1	10	***
	Evaluation			

765 PURE MATHEMATICS WITH MECHANICS

Assessment Objectives.

The abilities assessed in the examination covers a single area technique with application. Examination will test the ability of candidates to:

- Understand relevant mathematical concepts, terminology and notation.
- State accurately and use appropriate manipulation techniques.
- Apply combinations of mathematical skills and techniques in solving problems and to solve problems for which immediate solutions are not available but which demand a combined knowledge of more than one topic on the syllabus.
- Understand and apply the concepts of Mechanics and probability.
- Present mathematical work and communicate conclusions in a clear and logical way.

Note.

Candidates will be expected to be familiar with scientific notations for the expression of units, e.g. 3 ms^{-1} to mean 3 meters per second.

Detail Structure of the Examination.

The examination will consist of three papers. Questions will be in SI units.

Paper 1

A paper of $1\frac{1}{2}$ hours, on the whole syllabus which will carry one-fifth of the maximum mark. It will consist of fifty compulsory multiple choice questions, thirty-five from the core syllabus and fifteen from the syllabus for paper three. Answers will be marked on a multiple choice answer sheet to be provided. Calculating aids, for example, calculators and mathematical tables are not allowed in this paper.

Paper 2

A paper of $2\frac{1}{2}$ hours, on the core syllabus (Pure Mathematics) only, which will carry two-fifth of the maximum mark. It will consist of about ten questions with varying mark allocations per question which will be stated on the paper. Candidates shall be expected to attempt all the questions.

Paper 3

A paper of $2\frac{1}{2}$ hours, on the probability and Mechanics syllabus, which will carry two-fifth of the maximum mark and will consist of eight questions of which candidates will be required to answer all, each question carrying equal marks.

Paper 2 will be a common paper for both 765 Pure Mathematics with Mechanics and 770 Pure Mathematics with Statistics.

A booklet of mathematical formulae including statistical formulae and tables will be expected for papers 2 and 3.

The value of the Acceleration of free fall, g , will be 9.8 m s^{-2} except otherwise stated on the question paper.

The Syllabus

The syllabus has been introduced to bring together the “Modern” and “Traditional” approaches to Advanced Level Mathematics.

In drawing up the syllabus, attention has been paid to the content of Ordinary Level 575 Further (Additional) Mathematics syllabus, and it is assumed that candidates will have knowledge of the content of that syllabus. The syllabus for paper 2 contains a minimal “core” of Mathematics, the desirability of which has received very wide approval. In view of its importance, this “core” syllabus is examined without choice of questions both in paper 1 and in paper 2.

In any section of the syllabus, candidates will be expected to demonstrate that, as a proof of a result, a general proof is required, whereas a single counter-example will suffice to disprove a result. The emphasis will be on simple direct questions. Complicated questions involving excessive manipulations will not be set.

If a numerical answer is required, the question will specify, “to make many significant figures

or decimal places” otherwise the answer may be left in a form such as in $\frac{32}{27}$ or $\pi(\sqrt{3} + \sqrt{5})$.

The Core Syllabus - Pure Mathematics (Paper 2)

N°	SUB N°	Topic	Notes
1	1.0	Mathematical Statements, proof, logic, and sets	
	1.1	Some basic Logic.	If-then statements, logical equivalence, converse statements.
	1.2	Direct proof.	Forward-backward method, proof by contradiction.
	1.3	Sets and their representations.	Types and their notations.
	1.4	Power set.	
2	2.0	Binary Relations.	
	2.1	The concept of a binary relation in a set and from a set, E to another set F , ordered pairs.	
	2.2	Equivalence relations, equivalence classes.	Reflexive, symmetric, anti-symmetric and/or transitive relations.
	2.3	Other relations.	
	2.4	Relation of inclusion and total order on the set R .	
3	3.0	The concept of a function as a one-one or many-one mapping from R (or a subset of R) to R.	
	3.1	The domain and range (or image set) of a function.	
	3.2	Investigation of simple odd and even functions.	
	3.3	Composite functions.	Equations involving composite functions.
	3.4	The inverse of a one-one function.	
	3.5	Graphical and other representations of a function.	Manipulation of piecewise defined functions is expected.
	3.6	Graphical illustration of the relationship between a function and its inverse.	Simple algebraic functions only.

N°	SUB N°	Topic	Notes
	3.7	Periodicity of a function: The definition $f(x + a) = f(x)$ of a periodic function of period a , implies that a is the smallest positive constant having this property.	Sketching the functions within extended range and calculating the area under the curve.
	3.8	Injective, surjective and bijective mappings.	An understanding of the domain and range of a function is expected.
4	4.0	The use, manipulation and graphs of simple algebraic, trigonometric and exponential functions, and combinations of these functions	
	4.1	The use and properties of indices and logarithms. The definition $a^x = e^{x \ln a}$, $e^{x+y} = e^x e^y$	This shall include other exponential equations.
	4.2	Ability to sketch curves such as $y = x^n$, for integral and simple rational n , $ax^2 + by^2 = c$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	
	4.3	Knowledge of the effect of simple transformations as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x - a)$, $y = f(ax)$.	
	4.4	The relation of the equation of a graph to its symmetries.	
	4.5	The modulus function $ x $ and its graphs.	
5	The concept of a linear relation.		
	5.1	Reduction of relations such as $y = ax + b$, $y = ax^n$ $y = abx$, $y = ax^2 + b$, and $\frac{1}{x} + \frac{1}{y} = \frac{1}{a}$ to a linear form.	A knowledge of $Y = mX + C$ is expected. The use of logarithms and other methods in such cases is expected.
	6.0	Treatment of curves including curves with	

N°	SUB N°	Topic	Notes
6		simple parametric equations.	
	6.1	Tangents and normal to curves where x and y may be expressed parametrically as either algebraic or trigonometric functions.	Cartesian equations may be required as in 4.2.
	6.2	Areas under curves where x and y may be expressed parametrically as either algebraic or trigonometric functions	
	6.3	Loci using Cartesian or parametric forms: Knowledge of the geometrical properties of the circle is expected <i>but not of the parabola, ellipse or hyperbola.</i> The forms considered will include $(ct, \frac{c}{t})$, $(at^2, 2at)$, (at^2, at^3) and $(a \cos t, b \sin t)$.	Knowledge of • intersecting circles, • tangents and normal to the circles will be required.
	6.4	Curve sketching of curves given in either Cartesian or parametric forms.	
	6.5	Asymptotes in simple cases. (<i>Questions will not be set involving oblique asymptotes</i>).	
7	7.0	Vectors	
	7.1	Algebraic operations of the addition of two vectors and the multiplication of a vector by a scalar, and their geometrical significance.	Knowledge of direction ratios, direction cosines and direction vectors will be expected.
	7.2	The orthogonal unit vectors, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and the Cartesian components of a vector. Position vectors and displacement vectors.	A unit vector in a specified direction is expected.
	7.3	The equation of a line in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.	Knowledge of the parametric and Cartesian equations is expected.
	7.4	The scalar product of two vectors, its geometrical significance and algebraic properties.	Knowledge of the resolved part of one vector on another is expected.

N°	SUB N°	Topic	Notes
		The use of the scalar product for calculating the cosine of the angle between two lines.	
	7.5	-The angle between a line and a plane. -The angle between two planes. -The angle between two skew lines in simple cases.	Knowledge of $\mathbf{r} \cdot \mathbf{n} = -d$ is expected.
			Cartesian equation of a plane.
	7.8	The equation of a plane in the forms $\mathbf{r} \cdot \mathbf{n} = p$ and $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$, and the equivalent Cartesian forms. The vector product $\mathbf{a} \times \mathbf{b}$ or $ \mathbf{a} \mathbf{b} \sin \theta \mathbf{n}$	Perpendicularity of vectors. The geometric significance of $\mathbf{a} \times \mathbf{b}$ to the plane containing $\mathbf{a}$ and $\mathbf{b}$.
8	8.0	Circular functions	
	8.1	The use for a circle of $s = r\theta, A = \frac{1}{2} r^2 \theta$	
	8.2	Definition of the six trigonometric functions for any angle, knowledge of their periodic properties and symmetries.	Review of the unit circle.
	8.3	Sine and cosine formulae for a triangle.	
	8.4	The addition and product formulae; their use and applications. (Deduction of double and half-angle formulae from the addition formulae will be included).	
	8.5	The Pythagorean identities.	
	8.6	Expression of $a \cos \theta + b \sin \theta$ in the form $r \cos (\theta + \alpha)$ for $r > 0$, and α an acute angle.	
	8.7	General solution of simple trigonometric equations and particular solutions within given intervals to include: those of the form $a \cos \theta + b \sin \theta = c$ and those requiring either the use of the addition	This may also include equations from factor formulae.

N°	SUB N°	Topic	Notes
		formulae or the Pythagorean identities. (Involved trigonometric identity questions will not be set).	
	8.8	The approximations $\sin x \cong x$, $\tan x \cong x$, $\cos x \cong 1 - \frac{1}{2} x^2$	
	8.9	The inverse of trigonometric functions defined over suitable domains; for example: f^{-1} , where $f: x \mapsto \sin x$ ($x \in \mathbb{R}$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$).	
9	9.0	Simple cases of Permutations and Combinations	
	9.1	Successive operations in a row and in circular arrangements. (Independent situations).	Problems in simple restrictions will be set.
	9.2	Mutually exclusive situations.	
	9.3	Permutations of objects selected from a group. ${}_n P_r = \frac{n!}{(n-r)!}$	
	9.4	Arrangements of like and unlike objects.	
	9.5	Combinations. Selections of objects from a group ${}_n C_r = \frac{n!}{(n-r)!r!}$	
	9.8	Simple problems involving arrangements.	Include linear and circular arrangements.
	9.9	Simple problems involving selections.	
	9.10	The use of the binomial expansion of $(1+x)^k$ when k is a positive or negative integer and when k is rational and $ x < 1$ is also rational. (The complete expansion is required when k is a positive integer but only the first few terms will be considered as an approximation to $(1+x)^k$ when k is not a positive integer and $ x < 1$).	The validity of certain expansions will be required. Applications to approximations may be tested.

N°	SUB N°	Topic	Notes
			Knowledge of the constant term may also be required.
10	10.0	Polynomials with real coefficients, and their real factorization.	
	10.1	An understanding of the sign of the values of a polynomial expression -either algebraically or from a graph.	
	10.2	The factor theorem.	
	10.3	The remainder theorem.	Linear and quadratic remainders may be required.
11	11.0	The theory of quadratic functions: $f(x) = ax^2 + bx + c$, $a \neq 0$, and quadratic equations: $f(x) = 0$.	Sum of roots and products of roots and their manipulation with coefficients.
	11.1	Solution of the quadratic equation $f(x) = 0$.	Recognise and solve equations which can be transformed to quadratic equations.
	11.2	The sign of the value of $f(x)$. Graphs of $y = f(x)$ and $y = 1/f(x)$.	
	11.3	Maxima and minima.	
	11.4	Simple cases of symmetric functions of the roots of $f(x) = 0$ without solving the equation, and applications.	
	11.5	The set of values of expressions such as $\frac{px + q}{x^2 + 1}$.	
12	12.0	Simple algebraic inequalities and inequations	

N°	SUB N°	Topic	Notes
		including the use of the modulus sign.	
	12.1	Inequalities such as $a^2 + b^2 \geq 2ab$.	
	12.2	Inequalities such as $(x - \alpha)(x - \beta)(x - \gamma) > 0$.	Deduction of ranges from critical values.
	12.3	Inequalities such as $\frac{1}{x-a} + \frac{1}{x-b} > 0$	
	12.4	Inequalities such as $ x - a > k x - b $.	
	13.0	Rational polynomial functions	
	13.1	Decomposition when the degree of the numerator is less than the degree of the denominator.	Idea of improper fractions: linear, quadratic and linear repeated factors in denominators.
13	13.2	Decomposition when the degree of the numerator is greater than or equal to the degree of the denominator (<i>the degree of the denominator not greater than 3</i>).	
	13.3	Use of partial fractions in simplifying integration or differentiation.	
14	14.0	Sequences and series	
	14.1	Arithmetic progressions, arithmetic series and arithmetic means. The arithmetic mean. Sum of the first n terms.	Number patterns sequence of real numbers, the n th term. Limit of sequence, sequences diverging to $\pm\infty$. Bounded and monotone sequences.
	14.2	Geometric progressions, geometric series and geometric means. The Sum of the first n terms.	Convergence of geometric sequences, limit of sequences, sequences diverging to $\pm\infty$.

N°	SUB N°	Topic	Notes
	14.3	Summation of simple finite series, sigma notation	e.g. $\sum r, \sum r^2, \sum r(r+1)$.
	14.4	An intuitive idea of a limit and the convergence of the geometric series.	Sum to infinity of a geometric series ($ r < 1$).
	14.5	Use of partial fractions in the summation of series.	Method of difference in simple cases.
	14.6	Principles of mathematical induction.	
15	15.0	Differentiation.	
	15.1	Uniqueness of the limit; limit of sum, product and quotient. (Proofs not expected). Theorem; There exist a unique differentiable function defined on $\mathbb{R}$ such that $f'(x) = f(x)$, $f(0) = 1$. $e = \lim_{h \rightarrow 0} \frac{e^{h+1} - 1}{h}, f(x) = e^x \text{ or } \exp(x)$	$n \rightarrow \infty, \left(1 + \frac{x}{n}\right)^n \rightarrow e^x;$ $\frac{x^n}{n!} \rightarrow 0;$ $n^a x^n \rightarrow 0 \text{ for all } a, x < 1,$ $\text{As } x \rightarrow \infty, x e^{-ax} \rightarrow 0 \text{ for all } a > 0;$ $x^a e^{-x} \rightarrow 0, \text{ for all } a > 0$
	15.2	The derivative defined as a limit, differentiation from first principles.	
	15.3	The gradient of a tangent as the limit of the gradient of a chord.	
	15.4	Differentiation of standard functions such as: x^n , $\sin x$, $\cos x$, $\tan x$, $\arcsin x$, $\arccos x$, $\arctan x$, e^x , a^x , $\ln x$.	
	15.5	Differentiation of sums, products and quotients.	
	15.6	Differentiation of composite functions.	Chain Rule as a function of a function.
	15.7	Differentiation of inverse functions (limited to inverse trigonometric functions).	

N°	SUB N°	Topic	Notes
	15.8	Differentiation of functions expressed implicitly.	Two dimensions only.
	15.9	Differentiation of functions expressed parametrically.	Two dimensions only.
	15.10	Application of differentiation to rates of change, tangents, normal, maxima and minima, and points of inflexion. Curve sketching.	Rolle's theorem, Mean-value theorem, De L'hospital's rule, Increasing and decreasing test, first derivative test, the chart of signs, investigation of the equation $y' = ky$.
16	16.0	Integration and applications	
	16.1	Integration as the inverse of differentiation. Deduction of the integrals of standard functions from corresponding derivatives to include the integrals of x^n , $\frac{1}{x}$, e^x , $\sin x$, $\cos x$, $\frac{1}{\sqrt{1-x^2}}$.	
	16.2	Indefinite and definite integrals.	
	16.3	Decomposition into partial fractions as a technique of integration.	Linear and/or quadrate factors in the denominators.
	16.4	Use of linear and non-linear substitutions (Substitutions will be given except in the easiest cases).	Algebraic or trigonometric substitution will be set.
	16.5	Simple integration by parts. (Questions may be set which require more than one application of integration by parts).	Special forms are expected to be taught.
	16.6	Simple applications of integration to the calculation of plane areas.	To include area below the x-axis.

N°	SUB N°	Topic	Notes
	16.7	Simple applications of integration to the calculation of volumes of revolution.	Rotations completely about both axes.
	16.8	Simple applications of integration to the calculation of centroids and centres of mass.	Only those of plane figures will be considered.
	16.9	Approximations to definite integrals using the trapezium rule.	
	17.0	First order variable separable differential equations.	
17	17.1	Solution of simple first order differential equations with variables which can be separated.	Should involve trigonometric functions as well.
	17.2	Formation and solution of first order variable separable differential equations.	No prior knowledge of physical, chemical, biological or other laws will be required.
	17.3	Determination of arbitrary constants and sketching of members of families of solution curves.	Particular solutions.
18	18.0	Complex numbers	
	18.1	The Argand diagram. Geometrical representation of a complex number.	Including cube roots of unity: $1, \omega, \omega^2$
	18.2	Modulus and argument of a complex number.	Express in polar and exponential forms.
	18.3	De Moivre's theorem.	To derive the identities of the form $\sin(n\theta), \cos(n\theta), \tan(n\theta)$, express in powers of $\sin\theta, \cos\theta$ and $\tan\theta$ Expressing powers of sines / cosines in multiple angles.

N°	SUB N°	Topic	Notes
	18.4	Sum, product and quotient of a complex number.	Simple equations involving them with remainder and factor theorems, the functions $f(z)$.
	18.5	Use of conjugate complex numbers.	
	18.6	Geometrical representation of sums, products and quotients of complex numbers.	Simple loci are expected.
19	19.0	Location of the roots of any equation.	
	19.1	Determination of an interval within which a root of an equation lies, either graphically or by the use of the signs of $f(x)$.	
	19.2	Use of graphical methods to find an approximate value for the root of an equation.	
	19.3	Use of the Newton-Raphson process to find an approximate value for the root of an equation.	
	19.4	Use of any other iterative process (including the interval bisection) linear interpolation process to find an approximate value for a root of an equation. (An understanding of the convergence of an iterative process is expected).	
20*	20.0	Matrices	
	20.1	Solution of a system of linear equation (a minimum of three equations in the unknowns).	Their geometrical interpretation and solution structure.
	20.2	Conditions for the existence of a unique solution, no solution and infinity of solutions.	
	20.3	Definition of a matrix, the terms, element, row, column and order.	Types of matrices and algebra of matrices (equality, addition, subtraction and multiplication by a scalar); only addition is commutative.

N°	SUB N°	Topic	Notes
	20.4	Concept of elementary row and column operations.	Invertible matrices and proof of the uniqueness of the inverse, if it exists.
	20.5	Transpose of matrix.	$(A)^T(B)^T = (A^T B^T)$
	20.6	Determinant of a square matrix.	Factorisation of determinants
	20.7	Calculation of area and volume scale factors for transformations representing enlargements in two and three dimensions.	Linear transformation of points and lines will also be required.
	20.8	Inverse of a square matrix.	
	20.9	Matrix transformations in 2- and 3- dimensions.	Combined transformations.
	20.10	Invariant points and lines.	

Paper 3 (765) Mathematics with Mechanics

N ^o	SUB N ^o	Topic
21	21.0	Differentiation of a vector with respect to a scalar variable. For example, find $\mathbf{r}$ at time t seconds, given that $\mathbf{r} = [t^2\mathbf{i} + (\cos t)\mathbf{j} + e^{-t}\mathbf{k}]m$.
	21.1	Use of $\mathbf{r}'(t)$ as the velocity of a particle at time t seconds.
	21.2	Use of $\mathbf{r}''(t)$ as the acceleration of a particle at time t seconds.
22	22.0	Centres of mass of uniform rigid bodies.
	22.1	Determination of centres of mass using integration.
	22.2	Determination of centres of mass of composite bodies using symmetry and / or integration.
23	23.0	Force as a vector.
	23.1	The resultant of two coplanar forces.
	23.2	Moment of a force. (The moment of a force as a vector product).
	23.3	Couples.
	23.4	Simple cases of the equilibrium of particles and rigid bodies under coplanar forces. Emphasis will be placed on physical principles. <i>(Complicated problems that are mainly trigonometric puzzles will not be set).</i> The simple cases of applications of the basic conditions of equilibrium to uncomplicated systems only are required.
	23.5	Friction.
	23.6	Hooke's law.
24	24.0	Kinematics of a particle moving in a straight line.
	24.1	The use of the equations $\frac{dv}{dt} = f(v)$ or $f(t)$
	24.2	The use of the equations $v \frac{dv}{dx} = f(x)$ or $f(v)$
25	25.0	Velocity and acceleration as vectors.
	25.1	Relative velocity.
26	26.0	Kinematics of a particle in a plane. Velocity and acceleration when position is a function of time.
	26.1	Simple problems on projectiles.
	26.2	Motion in a circle with uniform speed.
	26.3	Angular speed.

N°	SUB N°	Topic
27	27.0	Newton's laws of motion.
	27.1	The setting up and solution of equations of motion for a particle in one or two dimensions under constant forces.
	27.2	Motion of connected particles moving in one dimension when the forces on each particle are constant and motion under a force which changes from one constant value to another.
	27.3	Uniform motion in a circle.
28	28.0	Momentum as a vector.
	28.1	The impulse-momentum principle.
	28.2	Conservation of linear momentum.
	28.3	Direct impact of elastic bodies. (The inequalities $0 \leq e \leq 1$ are expected to be known.)
29	29.0	Work, energy and power.
	29.1	Work, energy and the work-energy principle.
	29.2	Power; use of $P = Fv$.
30	30.0	Simple applications of elementary probability.
	30.1	Familiarity with simple experiments such as tossing coins, rolling dice or drawing cards from a pack of playing cards with or without replacement. "Possibility space" and "probability space" for such experiments.
	30.2	Conditional probability, $P(A B)$.
	30.3	Sum and product laws: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(B) \cdot P(A B)$
	30.4	The use of $P(A') = 1 - P(A)$.
	30.5	Independence of events.
	30.6	Mutually exclusive events.
	30.7	The distinction between mutually exclusive and independent events.
	30.8	An understanding and use of tree diagrams.

770 PURE MATHEMATICS WITH STATISTICS

AIMS:

In addition to the aims of Advanced Level Mathematics subjects, the specific aim of this syllabus which is not intended primarily for specialist as Mathematicians is to enable schools and colleges to provide courses which place special emphasis on evaluation in relation to scientific experimentation. Experimental work should form an integral part of the course.

Objectives

The objectives of the examination are to test the ability of all candidates to:

- i. demonstrate a confident knowledge of Mathematics and of the techniques of Pure Mathematics specified in the syllabus.
- ii. understand the principles of probability and statistics.
- iii. apply their knowledge of Mathematics to solve probability and statistics.
- iv. evaluate and explain clearly the significance of statistical calculations in contexts of scientific investigation.
- v. demonstrate understanding of their project work.
- vi. apply their knowledge of mathematics to problems which involve more than one topic of the syllabus.
- vii. write clear and accurate mathematical solutions.
- viii. demonstrate an understanding of the nature of proof and disproof in mathematics.

The Examination

The examination will consist of three papers. Questions will be set in SI units

Paper 1.

A paper of $1\frac{1}{2}$ hours on the whole syllabus will carry one-fifth of the maximum marks. It will consist of fifty compulsory multiple choice questions, thirty-five from the core syllabus and fifteen from the syllabus for paper three. Answers will be marked on a multiple answer sheet to be provided. Calculating aids, for example, calculators and mathematical tables may not be used in this paper.

Paper 2

A paper of $2\frac{1}{2}$ hours on the core syllabus only, will carry two fifths of the maximum marks. It will consist of about ten questions with varying mark allocations per question which will be stated on the paper. All these questions may be attempted.

Paper 3

A paper of $2\frac{1}{2}$ hours on the probability and statistics syllabus will carry two-fifths of the

maximum marks. It will consist of eight questions, of which candidates will be required to answer six, each question carrying equal marks.

Paper 2 will be identical to paper 2 of Pure Mathematics with Mechanics 765.

A booklet of mathematics formulae including statistical formulae and tables will be provided by the Examination Board for use with paper 2 and 3.

The Syllabus

The Core Syllabus: This portion shall be assessed using paper 2

The syllabus for this paper is identical to that of Paper 2 of syllabus 765 (see pages 5-15)

Paper 3 (770 only) Statistics

N ^o	Sub No	Topic
21	21.0	Elementary Probability
	20.1	Familiarity with simple experiments such as tossing coins, rolling dice or drawing cards from a pack of playing cards with or without replacement. "Possibility spaces" and "probability spaces" for such experiments.
	21.2	Conditional probability.
	21.3	Sum and product laws: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(B) \cdot P(A B)$.
	21.4	The use of $P(A') = 1 - P(A)$
	21.5	The distinction between two mutually exclusive and two independent events: $P(A B) = 0$ if and only if A and B are mutually exclusive. $P(A B) = P(A)$ if and only if A and B are independent.
	21.6	Empirical illustration of the laws in 20.5.
	21.7	Meaning and use of tree diagrams.
22	20.0	The idea of a random variable. Discrete probability distributions.
	22.1	The probability function $P(x) = P(X = x)$, for a discrete probability distribution.
	22.2	The expected value (the expectation) $E(X)$, the mode, median of a discrete random variable.
	22.3	The variance, $\text{Var}(X)$, of a discrete random variable.

N ^o	Sub No	Topic
	22.4	The use of the (cumulative) distribution function $F(x)$, where $F(x_0) = P(X \leq x_0) = \sum P(x)$, $x \leq x_0$.
	22.5	The uniform distribution and derivation of its mean and variance.
	22.6	The binomial distribution and derivation of its mean and variance.
	22.7	The geometric distribution.
	22.8	Mean of the sum or difference of two discrete random variables. Use of $E(X_1 \pm X_2) = E(X_1) \pm E(X_2)$.
	22.9	Variance of the sum or difference of two discrete random variables when the variables are independent: use of $\text{Var}(X_1 \pm X_2) = \text{Var}(X_1) + \text{Var}(X_2)$, when X_1 and X_2 are independent.
	22.10	The Poisson distribution and the use of experiments to illustrate the probability function $P(X = x) = \frac{e^{-m} m^x}{x!}$ and $E(X) = \text{Var}(X)$ (Knowledge of the exponential series is required).
	22.11	Use of the Poisson distribution as an approximation to the binomial distribution.
	22.12	Relationship between probability distribution and frequency distribution.
23	23.0	Continuous probability distributions.
	23.1	Definition of the probability density function, $f(x)$, and the use of $P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f(x) dx$.
	23.2	The use of the (cumulative) distribution function $F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(x) dx$.
	23.3	Determination of the mean, median, mode or quartiles of a specified continuous function.
	23.4	Determination of the variance of a specified continuous function.
	23.5	The exponential distribution including its mean and variance.
	23.6	The normal distribution: The use of tables to find probabilities for a normally distributed variable.

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	23.7	Empirical verification of the 2σ rule for a normal distribution.
	23.8	The use of the normal distribution as an approximation to the binomial distribution with application of the continuity correction.
	23.9	The use of the normal distribution as an approximation to the Poisson distribution with application of the continuity correction.
24	24.0	Samples and populations.
	24.1	Graphical representation of sample data. Frequency and cumulative frequency polygons for ungrouped and grouped sample data.
	24.2	The smoothing of a cumulative relative frequency polygon as an estimate of the population (cumulative) distribution function and the use of such polygons to find population medians and quartiles.
	24.3	The mean as a measure of location.
	24.4	The median as a measure of location.
	24.5	The mode as a measure of location.
	24.6	Weighted means (e.g. index numbers) as measures of location.
	24.7	Variance and standard deviation as measures of dispersion.
	24.8	Range and interquartile range as measures of dispersion.
	24.9	Combined mean and combined variance for two or more samples.
25	25.0	Sampling with or without replacement from a finite population
	25.1	Sampling distributions of statistics.
	25.2	Use of random numbers.
	25.3	Sampling of attributes
26	26.0	Sampling from an infinite population
	26.1	The distribution of sample means from an infinite population; the mean and standard error $\frac{\sigma}{\sqrt{n}}$ of this distribution.
	25.2	Central Limit theorem for large samples. (Empirical evidence only for the Central Limit theorem).
	25.3	Application to determining confidence limits for the mean, and for the difference between two population means.

N ^o	Sub No	Topic
	25.4	$\bar{X}$ For small samples, the sample mean $s^2 = \frac{\sum (x - \bar{x})^2}{(n - 1)}$ as unbiased estimates of the corresponding population parameters.
27	27.0	Hypothesis testing (for large samples only).
	27.1	The roles of the null and alternative hypotheses in significance testing.
	27.2	Significance testing of a hypothetical value for probability and for the means of Poisson distribution.
	27.3	Significance testing for a population means using a large sample (Using the Central Limit theorem).
28	28.0	Regression.
	28.1	Scatter diagrams.
	28.2	Simple linear regression, i.e. line of best fit with applications.
28	28.3	The distinction between the two regression lines y on x and x on y .
	28.4	The regression coefficient of y on x: $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$ (Candidates will not be expected to derive the equation of a regression line in full).

N ^o	Sub No	Topic
		The minimum sum of squares of the residuals.
29	29.0	Correlation.
	29.1	The product moment correlation coefficient.
	29.2	Kendall's rank correlation coefficient; its use and limitation.
	29.3	Spearman's rank correlation coefficient; its use and limitation. (The underlying principle of each coefficient should be known but the full derivation will not be expected. No correction for ties).